# Market Design

**Atila Abdulkadiroglu* and Lawrence M. Ausubel†**

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*Abstract*

Market design is the design of practical allocation mechanisms. In its relatively brief life, it has already become extremely influential. Theoretical developments have been successfully applied to important real-world markets and have touched many lives in positive ways; at the same time, new questions and methodological approaches motivated by practical policy concerns have been brought to the research frontier. Despite the risk of market design coming across as the forced marriage of two disparate literatures on auctions and matching, we demonstrate that this arises largely from their adoption of opposite extremes of the cardinal/ordinal utility spectrum. Many of the practical issues faced are common to both literatures, including: caps and quotas, package preferences, compact language, participation, commitment, timing, market size, product design, and secondary markets.

* Department of Economics, Duke University, Durham, NC 27708, USA. Email: atila.abdulkadiroglu@duke.edu

† Department of Economics, University of Maryland, College Park, MD 20742, USA. Email: ausubel@econ.umd.edu

# 1 Introduction

Market design emerged as a field in its own right in the 1990s. Its growing influence and success was recognized in the 2012 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel to Alvin E. Roth and Lloyd S. Shapley for the theory of stable matching and the practice of market design,[1] following earlier recognition given to some of the pioneers of auction theory and mechanism design.

*Market design* is the design of practical allocation mechanisms. It is done most successfully—one might well take the viewpoint that it can only be done successfully—when the designer has knowledge and control of the entire set of rules of the game. It assesses outcomes in relation to properties such as efficiency, stability and revenue optimization; it critiques processes in relation to properties such as robustness, voluntary participation and strategy proofness. It often studies how algorithmic details can influence the outcomes. It utilizes the tools of game theory and mechanism design, it considers participants' incentives, it accounts for domain-specific institutional details and constraints, and it looks at the feasibility of implementation.

The field was born out of two seemingly unrelated classics of the early 1960s. William Vickrey's 1961 "Counterspeculation, Auctions, and Competitive Sealed Tenders" initiated the game-theoretic analysis of auctions and foreshadowed many of the results of the subsequent 20 years of auction literature. David Gale's and Lloyd S. Shapley's 1962 "College Admissions and the Stability of Marriage" introduced the concept of stable allocations and formulated an enduring algorithm, establishing the modern matching literature. These landmark articles spawned two voluminous literatures that developed relatively independently over the subsequent half century.

Two subsequent developments led to the birth of market design out of these mainly theoretical exercises. The first was the design of the US Federal Communications Commission's auctions of licenses for radio spectrum (McMillan 1994, Cramton 1995, McAfee and McMillan 1996, Milgrom 2004). The second was Roth's (1984) discovery that the National Resident Matching Program (NRMP) had independently invented and had been using a variant of Gale and Shapley's deferred acceptance algorithm to match medical students with hospitals for their residency programs since the 1950s and Roth's later redesign of the NRMP.[2]

[1] See also Economic Sciences Prize Committee of the Royal Swedish Academy of Sciences (2012), Roth (2012), and Shapley (2012).

[2] During the same period, a number of other researchers also made influential contributions that helped to set the stage for the emerging field. Robert Wilson did extensive work on designing novel electricity tariffs and,

Given the pivotal role of spectrum auctions and the NPRM in the advent of market design, there is a danger of the new field being regarded as merely the union of auction theory and matching theory as applied to real-world problems. However, our viewpoint is that its scope ought to be viewed much more broadly. Market design is potentially practiced whenever the complete rules governing an aspect of an economic transaction can be specified and studied or improved. For example, while the health insurance market includes an array of complications over which no designer can have full control, the Affordable Care Act focuses in part on the adverse selection problem and a possible unraveling of the market: sick people will disproportionately buy insurance, many healthy people will not, and prices to the sick will increase accordingly; the higher prices may induce other healthy people to opt out, raising prices further.[3] It proposed the "individual mandate" as a complete solution. As such, the design of the Affordable Care Act may be viewed in much the same spirit as other market design projects surveyed below.

By contrast, the bargaining literature also addresses a pervasive economic problem. Nonetheless, most bargaining questions are not amenable to market design as currently practiced. The key consideration is that leading models, such as the alternating-offer bargaining game, ascribe a particular set of rules, whereas in reality, most bargaining is free-form and is not required to adhere to any clearly delineated set of rules. Bargaining studies are extremely important, but most do not fit naturally into the market design literature.

Further insight as to the scope of market design is provided by the seemingly contradictory perspectives of two pioneers. First, while the optimal trading rules of mechanism design theory may be highly sensitive to the probability distributions or functional forms governing a particular environment—"changing the environment requires changing the trading rule"—the trading rules commonly employed in the real world are not. The rules of prevalent markets "are not changed daily as the environment changes; rather, they persist as stable, viable institutions" (Robert Wilson, 1987, at pp. 36–37). This oft-repeated viewpoint, which has become so influential that it is referred to as the "Wilson doctrine," is often summarized as emphasizing that real markets employ rules that are independent of this type of detail and that perform robustly in a wide range of environments.[4]

particularly, wholesale electricity markets (Wilson 1993, 2002). Preston McAfee and John McMillan pursued a market design approach toward making government procurement more efficient (McAfee and McMillan 1988). Several researchers made contributions in which the ideas of auction theory were applied to the privatization of formerly state-owned enterprises (McAfee and McMillan 1987a, Maskin 1992).

[3] See, for example, D. M. Cutler and J. Gruber (2012) and D.M. Cutler et al (2012).

[4] See, for example, P. Dasgupta and E. Maskin (2000) at p. 347. Providing one possible interpretation of the Wilson doctrine, Bergemann and Morris (2005) pioneered a literature on robust mechanism design.

Second, other (often institutional) details of a market may be central concerns in designing a trading rule. "Market design involves a responsibility for detail, a need to deal with all of a market's complications, not just its principle features" (Alvin E. Roth, 2002, at p. 1341). As a result, theoretical insights from simple conceptual models regarding the general workings of markets are often insufficient for real designs. This calls for a discipline of market design that is almost synonymous with "economic engineering," where experimental and computational economics are often natural complements to game theory. The details highlighted by Roth often affect the existence and computation of plausible matching solutions. We would observe that any apparent conflict between Wilson's doctrine and Roth's attention to details disappears in most real matching markets, as these markets rely mostly on detail-free dominant strategy implementations.

The historical preponderance of market design work to date has involved auction theory or matching theory. Given that these two literatures developed largely separately, with their own distinct conventions and concerns, there is also the risk of market design coming across as the forced marriage of two disparate fields. However, our conscious perspective in this survey is that auctions and matching are actually two sides of the same coin. One of the distinguishing characteristics of the classical auctions and matching literatures is that they have largely adopted opposite extremes of the cardinal/ordinal utility spectrum. The traditional auctions literature takes the most cardinal of cardinal positions: agents have quasilinear cardinal utilities and everything can be priced. The traditional matching literature takes the extreme ordinal position: agents have purely ordinal rankings, without the possibility of any side payments. The differences in assumptions are often easily justified. The extreme cardinal view of auction models applies well to various prominent situations such as asset sales, leading to the implication that individually rational behavior under an optimal design yields social efficiency. At the opposite extreme, consider the problem of allocating limited numbers of seats at highly-sought public schools. Auctioning of school seats could achieve Pareto efficiency, yet social preferences for fairness preclude mere reliance on auction or pricing mechanisms, mandating an ordinal perspective.

Despite these differences, there is a strong sense of continuity between these two faces of market design. A growing body of works, beginning with Kelso and Crawford (1982) and Hatfield and Milgrom (2005), has demonstrated commonalities between auctions and matching and has offered a unified theoretical framework. In addition, one can identify auction markets where the auctioneer deviates from pure pricing mechanisms and matching markets where nonmonetary instruments play the role of prices. More importantly, we will demonstrate that many of the practical challenges faced in auction and matching markets are similar in nature.

Each of these faces of market design has a standard set of mechanisms that are frequently brought to bear on real-world problems. On the auction side, this survey will focus on clock auctions, a recent innovation on an old idea: the auctioneer announces prices, bidders respond with quantities, and prices are adjusted in relation to excess demand or supply until the market clears. On the matching side, this survey will begin with the deferred acceptance algorithm and the top trading cycles algorithm. However, as soon as one applies these standard mechanisms to real-world problems, one comes face to face with a variety of practical issues. Do societal objectives run counter to a simple aggregation of individual preferences, calling, for example, for caps or quotas? Are package preferences important to participants, creating a need for package bidding—or perhaps introducing complementarities so severe as to cause the performance of the standard mechanisms to go awry? How can participants' preferences be elicited in a simple and compact form? Is participation voluntary—and do the mechanisms determine allocations that induce players to participate voluntarily? Do participants commit to their promises, and how can mechanisms cope with commitment problems? Is the timing of offers conducive to efficient trade? Can the size of the market be enlarged so as to mitigate market power and thereby improve performance?

Any literature survey is limited by space, and it further needs to narrow its focus if it attempts to expound a coherent perspective. As a result, we apologize to our colleagues for omitting discussion of major strands of the relevant literature. For greater completeness, we refer the reader to other extensive surveys of auctions, matching and related literatures, including McAfee and McMillan (1987a), Roth and Sotomayor (1990), Milgrom (2004), Klemperer (2004), Cramton, Shoham and Steinberg (2006), Nisan, Roughgarden, Tardos and Vazirani (2007), Krishna (2012), McAfee and Vassilvitskii (2012), Jackson (2013), Vulkan, Roth and Neeman (2013), and Loertscher, Marx and Wilkening (2015).

Our survey is structured as follows. This Section 1 introduces the field of market design. Section 2 looks at the auction side of market design through a review of clock auctions. Section 3 examines the matching side of market design through the lens of stability, efficiency and incentives. Section 4 considers in detail the cardinal/ordinal dichotomy, the comparison of mechanisms, and the questions posed two paragraphs above: a common set of issues that recur in auction and matching applications. Section 5 speculates on areas and issues for future research. Section 6 concludes.

# 2 The Auction Side of Market Design: Clock Auctions

## *2.1 Introduction*

A first wave of auction research could be said to begin with the seminal 1961 article by William Vickrey, which led to a formidable body of research by pioneers including Robert Wilson, Edward Clarke, Theodore Groves, Paul Milgrom, Robert Weber, Roger Myerson, Eric Maskin and John Riley. This first wave of research, focusing on game-theoretic analyses of single-item auctions, concluded in the mid-1980's. By this time, there was a widespread sense that auction theory had become a relatively complete body of work with very little remaining to be discovered. See McAfee and McMillan (1987a) for an excellent review of the first wave of auction theory.

However, the perception that auction theory was complete began to change following two pivotal events in the early 1990s: the Salomon Brothers scandal in the US Government securities market in 1991; and the granting of spectrum auction authority to the Federal Communications Commission (FCC) by the US Congress in 1993. The aftermaths of these two episodes demonstrated that our systematic understanding of auctions and bidding had been largely confined to single-item auctions, while many or most of the empirically important applications of auctions involve a multiplicity of related items. At the same time, each episode underscored that the existing base of knowledge that could be applied in the field was surprisingly limited. Both episodes fostered the active involvement of academic economists and game theorists in the design of practical allocation mechanisms.

Thus, in these two events lay the seeds of a second wave of auction research—a market design literature—which emerged in the mid-1990s and continues today. The substantive emphasis of this literature has shifted significantly from single-item auctions to multiple-item auctions. But, perhaps more fundamentally, the recent literature has elevated the appreciation of practical design issues and research findings that can be taken to the field.

If a book were written reviewing the auction side of market design, chapter one would undoubtedly focus on the advent of the FCC spectrum auctions. It is possible to cite some work prior to the FCC auctions as consistent with our modern notion of market design. For example, in his seminal 1961 article, Vickrey suggested that the standard Dutch auction used in Netherland's flower auction be changed from a first-price to a second-price format,[5] and he proposed what is now known as the multi-unit Vickrey auction.[6]

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[5] Vickrey (1961, p. 23) effectively proposed a redesign of the Dutch auction: "On the other hand the Dutch auction scheme is capable of being modified with advantage to a second-bid basis, making it logically equivalent to the second-price sealed-bid procedure suggested above on page 20. As presently practiced, speed is achieved by having a motor-driven pointer or register started downward from a prohibitively high price by the auctioneer; each bidder may at any time press a button which will, if no other button has been pushed before, stop the register, thus indicating the price, flash a signal indicating the identity of the successful bidder, and disconnect all other buttons, preventing any further signals from being activated. There would be no particular difficulty in modifying the apparatus so that the first button pushed would merely preselect the signal to be flashed, but there would be no overt indication until the second button is pushed, whereupon the register would stop, indicating the price, and the signal would flash, indicating the purchaser."

[6] Vickrey (1961, pp. 10–13) proposed a sealed-bid auction format for homogeneous goods in which the goods are allocated to the highest bidders, and with the following novel pricing rule. If a bidder wins $k$ units, the bidder pays the highest rejected bid by any of its opponents for its $k^{th}$ unit, the second highest rejected bid by any of its opponents for its $(k-1)^{st}$ unit, …, and the $k^{th}$ highest rejected bid by any of its opponents for its first unit. In private

By the same token, two leading early contributions to combinatorial auctions were directed toward the practical allocation and pricing of airport landing slots (Rassenti, Smith and Bulfin 1982) and a NASA space station (Banks, Ledyard and Porter 1989), and some other early work was directed toward electricity markets (Wilson 1993, 2002), government procurement (McAfee and McMillan 1988), and privatization (McAfee and McMillan 1987a, Maskin 1992). Nevertheless, at the time, the early FCC auctions were the most influential episode to date in which economists and game theorists had engaged in the deliberate design of auction markets.

In FCC Auction #1, the Nationwide Narrowband Auction of July 1994, economists launched one of the most versatile auction techniques in the market design toolbox: the *simultaneous multiple round auction* (SMRA).[7] The SMRA is essentially an electronic implementation of the "silent auction", familiar from charity fundraisers, in which all of the items are offered simultaneously and bidders can successively raise each other's bids until the auction closes. However, it includes two critical innovations. First, an activity rule is enforced; bidders are required to engage in at least a minimum level of bidding in each round, in order to retain the right to bid for similar quantities of items in subsequent rounds. Second, the auction continues until no new bids are placed, rather than closing at a fixed ending time. Activity rules avoid the spectacle, in a silent auction ending at 9 pm, of hardly any bids being placed earlier than 8:55 pm. They are critical for transparency and are discussed further in Section 4.8. The stopping rule, under assumptions of substitute goods and non-strategic bidders, leads to a Walrasian equilibrium outcome (Milgrom 2000). Excellent accounts of the early FCC auctions are provided by McMillan (1994), McAfee and McMillan (1996), Cramton (1995) and Milgrom (2004).

The FCC auctions have been remarkably influential, both in their own substantive right and as the first instance when auction theorists served as market designers. To avoid repetition with the earlier accounts and to provide a fresher look, our current survey will instead focus on what might be considered as chapter two of the book on the auction side of market design: the development and introduction of clock auctions.

## 2.2 Clock auctions

By a *clock auction*, we mean an auction format in which the auctioneer announces prices and bidders respond with quantities, with iterative price adjustment until the market clears. In contrast to a traditional English auction at Sotheby's or Christie's, an internet auction at eBay or the FCC's SMRA, it is *not* the

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values settings in which bidders have weakly decreasing marginal utilities, truth-telling is a weakly dominant strategy of the multi-unit Vickrey auction.

[7] The SMRA is generally credited to Paul Milgrom, Robert Wilson and Preston McAfee.

bidders who successively propose prices. Rather, the auctioneer paces the movement of price, and bidders indicate quantities. A clock auction may involve a single item,[8] a single category of homogeneous goods,[9] or multiple categories of goods.[10] A clock auction may be ascending or descending, or prices may be permitted to move in both directions.

The earliest progenitor of the modern clock auction is the work of Walras (1874), who described a process in which a fictitious auctioneer names prices, bidders respond with quantities, and prices are adjusted upward or downward in relation to the quantity of excess demand. The Walrasian auctioneer—and the associated price adjustment process ("Walrasian tâtonnement")—was intended as a hypothetical device explaining the convergence to market-clearing prices. The construct was formalized by Samuelson (1941). Classical results within general-equilibrium frameworks include the global stability of Walrasian tâtonnement under the assumption of gross substitutes (Arrow, Block and Hurwicz 1959), and counterexamples (in non-substitutes environments) for which Walrasian tâtonnement fails to converge from any starting point other than the equilibria itself (Scarf 1960). Elegant reviews of the classical literature on tâtonnement stability are provided by Varian (1981) and Hahn (1982). A series of articles, beginning with Kelso and Crawford (1982), developed similar results within game-theoretic frameworks and with discrete goods—see Section 2.3.

However, in the modern era of market design, the Walrasian auctioneer has been reconceptualized as the basis for a real trading institution. Instead of a fictional auctioneer serving as a metaphor for a market-clearing process, a computerized auctioneer announces prices, accepts bids of quantities and adjusts prices. Perhaps the earliest appearance of clock auctions in market design was in the 1992 *Joint Report on the Government Securities Market*, the official response to the Salomon Brothers scandal. The *Joint Report*, discussing the sale of homogeneous government securities, proposed replacing the Treasury's sealed-bid auctions with uniform-price ascending clock auctions. This proposal was never implemented. However, less than a decade later, in 2001, the Electricité de France (EDF) Generation Capacity Auctions had become the first large real-world implementation of a simultaneous ascending clock auction. By the time of this writing, in 2016, the clock auction format has become commonplace for subject matter as diverse as electricity, natural gas, offshore wind energy leases, environmental allowances, rough diamonds, domain names, and spectrum.

[8] See, for example, Milgrom and Weber (1982).

[9] See, for example, Ausubel (2004).

[10] See, for example, Ausubel (2006).

## 2.3 Clock auctions with non-strategic bidders

One of the key classical results underlying clock auctions is due to Arrow, Block and Hurwicz (1959). Suppose that goods are gross substitutes in the sense that, if the prices of some goods increases while the prices of all other goods remain unchanged, then demand will not decrease for any of the goods whose prices have remained unchanged. Further assume that the goods are perfectly divisible and that demands are single-valued and continuous. The Walrasian tâtonnement process is given the literal interpretation that prices are adjusted as a differential equation based upon a continuous, sign-preserving function of excess demand. That is, the price is increased (decreased) for any good when there is positive (negative) excess demand and the price remains constant when demand exactly equals supply.

The gross substitutes condition implies the global stability of equilibrium in the sense that, starting from any price vector, the Walrasian tâtonnement process converges to the unique Walrasian equilibrium (Arrow, Block and Hurwicz 1959). Further, if every good has excess demand at the initial price vector, then the convergence is monotonic. The intuition is that, starting as such, the price adjustment path never exhibits excess supply. Suppose to the contrary that it did, and consider the first good to enter a state of excess supply. By continuity, at the moment it occurs, demand and supply are in balance for this good (while no other good is in excess supply). Consequently, the price of this good is unchanging, while all other prices are non-decreasing. By gross substitutes, excess demand for this good (weakly) increases, a contradiction. For the more restrictive case (but standard in the auction literature) where bidders have quasilinear utility functions, the substitutes condition can be somewhat weakened. Suppose merely that each bidder's utility is quasilinear in money and that the utility for goods is given by a strictly concave function. Then global stability of equilibrium still holds (see, for example, Varian, 1981, p. 102).[11]

More recent research has considered goods that are discrete rather than perfectly divisible. Such goods immediately violate the assumption that demands are single-valued. Quasilinear bidders with unit demands—a special case of substitutes—were investigated by Milgrom and Weber (2000), Demange, Gale and Sotomayor (1986), and McCabe, Rassenti and Smith (1990), while bidders with multi-unit demands and substitutes preferences were studied by Kelso and Crawford (1982), Gul and Stacchetti (1999, 2000), Milgrom (2000, 2004), Ausubel and Milgrom (2002), Ausubel (2005, 2006) and Milgrom and Strulovici (2009). Counterparts to the divisible-goods results hold: the set of Walrasian price vectors has a lattice structure and, if every good has excess demand, the Walrasian tâtonnement process converges monotonically to the lowest Walrasian price vector.

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[11] Consider the function $L(p) = p \cdot S + \sum_{i=1}^{n} V_i(p)$, where $V_i(p)$ is the indirect utility function of bidder $i$. Then it can be shown that $L(p)$ is a Lyapunov function for the dynamical system; in particular, $L(\cdot)$ decreases monotonically along the Walrasian tâtonnement path, and $L(\cdot)$ is minimized at the unique Walrasian equilibrium.

## 2.4 Clock auctions with strategic bidders

The results of Section 2.3 assumed that bidders are non-strategic price-takers. This assumption is justified if all bidders are of measure zero; but, otherwise, bidders' actions affect prices. Much of the intuition for how strategic behavior may affect outcomes can be obtained from examination of the sealed-bid, uniform-price auction for a homogeneous good with (weakly) diminishing marginal values. With non-strategic bidding, the uniform-price auction generates a Walrasian equilibrium outcome. However, if any bidder has market power, consider its optimal bidding strategy. Sincere bidding is weakly dominant for a first unit, as in the second-price auction for a single item: if a bidder's first bid determines the clearing price, then it is rejected, and the bidder wins zero units. However, the bidder's second and all subsequent bids may determine the price paid for the first unit, providing an incentive for *demand reduction*. The extent of bid shading increases in the number of units, since the number of inframarginal units whose price may be affected increases. At the same time, the allocation rule in the auction has the effect of equating marginal bids, but a large bidder will likely have shaded its marginal bid more than a small bidder, so the large bidder's marginal value is probably greater than the small bidder's. Consequently, except for two non-generic cases—when bidders have unit demands, and when there is a pure common value for the good—equilibria of the sealed-bid, uniform-price auction are inefficient (Ausubel, Cramton, Pycia, Rostek and Weretka 2014).

Meanwhile, the uniform-price clock auction inherits the demand reduction and inefficiency of the uniform-price sealed-bid auction and, indeed, the dynamic nature of the auction may exacerbate the demand reduction. An extreme case (and, thus, a cautionary note) is provided by an alternating-offer representation of a clock auction with complete information. Often, the unique subgame perfect equilibrium will have the first player reduce its demand to slightly more than half of the supply and will have the second player reduce its demand to the residual supply. Thus, the auction may clear at the lowest possible price, essentially independent of the level of demand (Ausubel and Schwartz 1999).[12]

The above considerations motivated Ausubel (2004), which is perhaps the first treatment of clock auctions in the mode of modern market design. Ausubel observed that the literature has provided two fundamental prescriptions guiding effective auction design. First, an auction should be structured so that the price paid by a player—conditional on winning—is as independent as possible of her own bids (Vickrey 1961). Second, it should be structured in an open fashion that maximizes the information made available to each participant at the time she places her bids (Milgrom and Weber 1982). In single-item environments, these dual prescriptions are often taken to imply the desirability of the English auction and

[12] See also Brusco and Lopomo (2002).

to explain its prevalence. However, for auctions of multiple items, no one had combined these two insights and taken them to their logical conclusion.

Ausubel treated the case of homogenous goods, and proposed an alternative auction design. As in a standard clock auction, the auctioneer announces prices, bidders respond with quantities, and the price is incremented if there is excess demand. However, unlike standard tâtonnement processes, bids made prior to the attainment of market clearing could still affect payments. After each round of bidding, the auctioneer checks whether any bidder has "clinched" one or more items, meaning that the bidder is guaranteed to win the items no matter what subsequently occurs. With homogeneous goods, this simply requires asking whether the aggregate demand of the bidder's opponents is strictly less than the supply—if so, the bidder has clinched the difference. The auction then proceeds, as in a standard clock auction, except that the bidder will be allocated the clinched items at the current price, not at the ultimate clearing price (which may be higher).

Straightforward bidding by all bidders is an equilibrium of the Ausubel auction. In addition, in a specific formulation of the auction game that includes incomplete information and a full support assumption, straightforward bidding is the unique outcome of iterated elimination of weakly dominated strategies. The explanation is that, just as the English auction is a dynamic game whose collapsed (static) representation is the second-price sealed-bid auction, the Ausubel auction is a dynamic game whose collapsed (static) representation is the multi-unit Vickrey auction. Truthful bidding is an equilibrium strategy in the second-price sealed-bid auction and the multi-unit Vickrey auction, and the corresponding dynamic auctions inherit this desirable property.

The analogous design can also be specified for a reverse auction to buy a homogeneous good. (Since the auction seeks to procure, the price clock is now descending, rather than ascending.) Again, the auctioneer checks at every price whether any items have been "clinched"; in the reverse auction setting, this requires evaluating the feasibility of meeting the buyer's demand without purchasing at least a particular quantity from a given bidder. Newly-clinched items are paid the current price, rather than the ultimate clearing price (which may be lower).

Ausubel (2006) proposes an efficient clock auction design for heterogeneous goods when bidders have substitutes preferences. Bikhchandani and Ostroy (2002, 2006) and De Vries et al. (2007) recast this approach by formulating the allocation problem as a linear program, formulating a primal-dual algorithm for this linear program, and interpreting the algorithm as a dynamic auction. Perry and Reny (2001, 2002) study more general homogeneous goods environments with interdependent values and find efficient designs for such environments. Dasgupta and Maskin (2000) define a sealed-bid auction procedure designed to attain efficiency with heterogeneous goods.

One of the most conspicuous extensions of this approach is made in the reverse auction component of the FCC's incentive auction (see Section 5.5). There, a television station offering to relinquish its broadcast rights becomes "frozen" when it is no longer feasible to assign this station to its pre-auction broadcast band while clearing the desired quantity of spectrum, given the current assignments of other stations (Milgrom, Ausubel, Levin and Segal 2012). In other words, "frozen" corresponds closely to "clinching" in a more conventional descending clock auction. However, in the incentive auction design, broadcast rights can become "unfrozen" if the auction progresses to a further stage in which a smaller quantity of spectrum is cleared.

## 2.5 Real-world implementations of clock auctions

Perhaps the earliest suggestion of the clock auction as a practical market design was in the 1992 *Joint Report on the Government Securities Market*, jointly authored by the US Department of the Treasury, the Securities and Exchange Commission, and the Board of Governors of the Federal Reserve System as the official response to the 1991 Salomon Brothers scandal in the government securities market. (Salomon apparently violated a 35% limit on the quantity than any participant could bid for or win in any Treasury auction. In one remarkable instance, Salomon, on behalf of itself and two customers, bid for and received approximately 86% of a two-year Treasury note being auctioned, with the apparent intent of engineering a "short squeeze" of the security.[13]) The *Joint Report* first weighs the merits of the sealed-bid, pay-as-bid auction (which was used at the time) versus the sealed-bid, uniform-price auction (which was ultimately adopted for selling all US government securities in 1998). Then, the *Joint Report* goes on to present a more intriguing possibility:

> Irrespective of whether the single- [uniform-] price, sealed-bid auction would prove superior to the current practice, the Agencies believe that there is an auction technique that may be superior to both types of sealed-bid auction techniques discussed above. This is an ascending-price, open auction system, which will be feasible for the first time once the auctions are automated. Auction theory suggests that, in general, Treasury revenue would not suffer, and indeed might increase, in the switch to an open, ascending-price system.[14]

[13] *In the Matter of John H. Gutfreund, Thomas W. Strauss, and John W. Meriwether*, Securities Exchange Act of 1934, Release No. 34-31554, December 3, 1992, p. 7, and US Department of the Treasury et al. (1992), pp. 14-15.

[14] US Department of the Treasury et al. (1992), pp. 14-15.

The *Joint Report*'s proposal seems to have been motivated by two considerations. First, "the open nature of the bidding, along with the single price outcome, should eliminate the 'winner's curse'." Second, "an open auction system allows participants to react to surprise bids, turning market forces against attempts at market manipulation. Entities attempting to corner this type of auction are effectively forced to disclose their intentions to their competitors … . Hence, the would-be market manipulators may fail to corner the security or, at the least, find it a more expensive proposition." [15]

The proposal was never implemented. Instead, later the same year, a Treasury auction experiment was launched, in which the US Treasury would sell all of its 2-year and 5-year Treasury notes by a sealed-bid, uniform-price auction, while continuing to sell all government securities of other durations by a sealed-bid, pay-as-bid auction. This experiment continued for six years, with relatively inconclusive results,[16] whereupon the Treasury shifted all auctions to the uniform-price format in November 1998.

Almost a decade would pass from the *Joint Report* until the first commercial implementation of a simultaneous clock auction. This came in September 2001, when Electricité de France (EDF) was required by the European Commission to offer 6 GW of electricity generating capacity, referred to as virtual power plants (VPPs), to competitors as part of the regulatory *quid pro quo* for an acquisition.[17] The EDF Generation Capacity Auctions were simultaneous clock auctions. A total of 10 or more VPP contracts were offered in two groups: base-load and peak-load products. Within each group, a variety of durations were offered: 3 months, 6 months, 12 months, 24 months and 36 months; all with the same starting date. All durations within a group were treated equivalently for purposes of clearing, since the sale of any of these contracts would put the same amount of electricity capacity into competitors' hands during the initial quarter. However, to the extent that the 3-month product was sold, an equivalent quantity would need to be offered in the next auction, whereas to the extent that the 12-month product was sold, an equivalent quantity would not need to be offered until four quarterly auctions later, etc. As such, the prices of the base-load and peak-load products were treated as two independent prices, while the prices of different durations within the same product were linked according to a predetermined relationship derived from market-based value differences (Ausubel and Cramton, 2004, 2010a).

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[15] US Department of the Treasury et al. (1992), p. 15.

[16] Malvey, Archibald and Flynn (1995) and Malvey and Archibald (1998).

[17] EDF, the dominant electric utility in France, sought in 2000 to acquire a joint controlling stake in Energie Baden-Württemberg AG (EnBW), the fourth largest electric utility in Germany. Since EDF would be gaining joint control of one of the potential competitors particularly well placed to enter the French market, the European Commission wished to make available to other potential entrants a significant quantity of France's generating capacity. At the same time, given that the large majority of EDF's base-load power plants were nuclear and given EDF's success with nuclear power, the regulator recognized the undesirability of physical divestment. Consequently, the regulator and EDF agreed on a *virtual* divestment of 6 GHz of French electricity capacity: virtual power plant auctions.

About 100 virtual power plant auctions were conducted in Europe from 2001–2014. EDF itself conducted 42 quarterly auctions through 2011, when EDF's acquisition was undone and the auction obligation was lifted. The basic mechanism was replicated by other European utilities, including Belgium's Electrabel (seven auctions in 2003–2005), Denmark's DONG Energy (35 auctions in 2005–2014), and Spain's Endesa and Iberdrola (seven auctions in 2005–2007). A similar structure was used in the Texas Capacity Auctions (about 20 auctions in 2002–2006).

Just as standard clock auctions are useful tools for selling goods, "reverse" clock auctions can be used for procurement. In 2002, two large descending clock auctions were implemented. In the first, the New Jersey Basic Generation Service (BGS) Auctions, descending clock auctions were utilized by four electric utilities to procure default service for consumers in the state of New Jersey (Loxley and Salant 2004). As of this writing, in 2016, the New Jersey BGS Auctions continued to be conducted on an annual basis. In the second, DEFRA (at the time, the UK environmental regulator) used a descending-clock auction to allocate £215 million in incentive payments for emission reductions as part of the initiation of the UK's emission trading scheme. As such, this was also the world's first auction for greenhouse gas emission reductions.

In subsequent years, dynamic clock auctions have been proposed for a variety of other applications and have been introduced into a variety of other sectors. All electricity generating capacity in the six-state New England region since 2008 has been procured using annual descending clock auctions (Ausubel and Cramton, 2010b) and a similar capacity market began to operate in the UK in late 2014. In the environmental sector, Australia's Department of Climate Change and Energy Efficiency proposed the adoption of ascending-clock auctions for greenhouse gas emission allowances, but plans for Australian carbon auctions were put on hold after a change in government.[18] For natural gas, the unfortunately named "gas release programme" auctions— gas sector counterparts of VPP auctions—were implemented utilizing ascending clock formats in Germany, Austria, France, Hungary and Denmark. Clock auctions have also been used for gas storage auctions in France. In rough diamond markets, there has been a noteworthy shift from opaque to transparent sales, as DeBeers' traditional model of take-it-or-leave-it offers for "boxes" of diamonds has begun to be swept aside by ascending clock auctions (Cramton,

[18] For the proposed adoption of ascending clock auctions for carbon units, see Government of Australia, Draft Clean Energy (Auction of Carbon Units) Determination 2013, Policies, Procedures and Rules for Auctioning Carbon Units, Explanatory Note, 26 March 2013 at http://climatechange.gov.au/sites/climatechange/files/files/reducing-carbon/1-auction-determination-explanatory-note-pdf.pdf. However, the ruling Labor Party was defeated in 2013 elections, substantially on the unpopularity of the "carbon tax", and Australia's carbon pricing mechanism was eliminated effective July 1, 2014; see http://www.cleanenergyregulator.gov.au/Carbon-Pricing-Mechanism/Carbon-pricing-mechanism-repeal/Pages/default.aspx.

Dinkin and Wilson 2013). After the financial collapse of 2008, clock auctions were proposed and tested in connection with the Troubled Asset Relief Program (TARP), but not adopted (Ausubel et al. 2013).

In the area of telecommunications spectrum, India has utilized ascending clock auctions for all of its spectrum auctions, beginning in 2010. Clock auctions underpin the combinatorial clock auction (CCA), which has recently established itself as one of the leading formats for spectrum auctions worldwide—see Section 4.4. The FCC's Incentive Auction Program, described in Section 5.5, proposes to use clock auction formats for both its reverse auction and forward auctions.

## 2.6 Discrete rounds and intra-round bidding

Economists' classical formalization of Walrasian tâtonnement imagined auctions in continuous time—price adjustment was modeled by a differential equation. However, practical market designs mostly use discrete rounds, rather than continuous bidding, for two important reasons. First, communication is rarely so reliable that bidders would be willing to be exposed to a continuous clock in a high-stakes environment. Bidders would find it unsatisfactory if price clocks advanced beyond their willingness to pay because of brief communication glitches. Discrete rounds are robust to communication problems. Second, discrete rounds improve price discovery by giving bidders the opportunity to reflect between rounds. Bidders need time to incorporate information from prior rounds into revised bidding strategies (Ausubel and Cramton 2004).

"Continuous" bidding is most commonly seen in Dutch auctions and in Internet auctions such as eBay. The former can be seen, for example, in Amsterdam's flower auction, in many fish auctions, and in US tobacco auctions. Bidding in Dutch auctions is usually on-site (avoiding communication difficulties). The Dutch auction is often used in place of the strategically equivalent sealed-bid first price auction (Vickrey 1961) when speed and high throughput are paramount, for example when goods are low in value and highly perishable. Consumer-oriented auctions such as eBay are generally for low-value items and often include a consumer search element and a significant entertainment component. Bids can then be submitted at any time during the fixed duration of the auction (for example, a one-week period) but bid-sniping is prevalent—see Section 4.8.

When discrete bidding rounds are used, an important issue is the size of the bid increments or decrements. Larger steps enable the auction to conclude in fewer rounds, but they potentially introduce inefficiency from the use of a coarse price grid. Larger steps also introduce incentives for gaming as a result of the expanded importance of ties and rationing rules. But using small steps especially in auctions with many products can vastly increase the number of rounds and, hence, the time required to complete the auction. A shorter auction reduces bidders' participation costs as well as their exposure to price movements during the auction. This is especially relevant for products (e.g., in energy) in which there are active secondary markets, where the underlying price movements could easily exceed the step sizes in the auction.

Fortunately it is possible to capture nearly all of the benefits of a continuous auction and still conduct the auction in a limited number of rounds, using *intra-round bids*. In a traditional clock auction, the price may increase from say $90 to $100 in a round, but the bidder is only able to express the quantity it desires at $90 and at $100. With intra-round bids, the bidder is asked to express a demand curve at all prices between $90 and $100. The auctioneer still paces the rate of price increase, but is able to avoid the inefficiency associated with a coarse price grid. More generally, with multiple goods, the bidder is asked to express its demands at all prices along the line segment from the start-of-round price vector to the end-of-round price vector. Intra-round bids were a design innovation in the EDF virtual power plant auctions and, since then, virtually all clock auctions used in practice have utilized intra-round bidding.[19]

# 3 The Matching Side of Market Design

## 3.1 Two-sided matching

Most matching models involve two sides to the market: workers seeking jobs and firms seeking to fill positions; students seeking school slots and schools needing to fill the slots; patients in need of organ transplantation and donors willing to donate organs; and so on. The simplest model, known as the marriage model, involves a set of men and a set of women.[20] Agents on one side of the market rank the agents on the other side and the option of remaining single in a strict preference order. Each agent can be matched with at most one agent from the other side. A matching of men to women is said to be *stable* if every agent prefers his or her match to being single and there is no unmatched pair of men and women such that they prefer each other to their match. A stable matching always exists in this market (Gale and Shapley 1962). Gale and Shapley proved this existence result via their celebrated *deferred acceptance* algorithm:

> Every man proposes to the woman he likes the most. Every woman who receives a proposal from at least one man keeps the proposal of the man she likes the most among her proposers and rejects the remaining proposals. In the following stages of the algorithm, every man whose proposal is

[19] The principal exceptions have been combinatorial clock auctions (CCAs), described in Section 4.4. In place of intra-round bids, the CCA has included a supplementary round, which permits bidders to name their own prices outside of the auctioneer-selected price grid.

[20] The Gale and Shapley (1962) model is not formulated as such out of any preference for defining a marriage as a union of one man and one woman, but on account that the existence of a stable matching is not guaranteed when marriage consists of pairs from a single set of agents. Examples where there is non-existence of stable matchings can be constructed with as few as three agents in the “roommates problem”.

rejected proposes to the woman he likes most among those who have not rejected him. The proposals and rejections continue until there are no more rejections.

This algorithm terminates and finds a stable matching in finite time. There may be more than one stable matching. For example, a distinct stable matching can potentially be achieved when the roles of men and women in the algorithm are interchanged (i.e., when women propose to men instead). Although conflict of interest among agents on one side of the market may exist—for example, all men may prefer the same woman the most—their interests are aligned over the set of stable matchings: All men like a stable matching at least as well as another if and only if all women like the latter at least as much as the former (Knuth 1976). In fact, the set of stable matchings forms a lattice (a result attributed to Conway; see Roth and Sotomayor 1990) so that, for example, men prefer the outcome of Gale and Shapley's man-proposing algorithm to any other stable matching (Gale and Shapley 1962).

From a market designer's point of view, preferences are market participants' private information. When agents on both sides of the market are strategic, there is no stable matching mechanism that is also strategy-proof, i.e., makes truthful reporting of preferences a dominant strategy for every player (Roth 1982). However, when, and only when, agents on one side of the market can be matched with at most one agent from the other side, the deferred acceptance algorithm in which these agents propose makes truth telling a dominant strategy for them (Dubins and Freedman 1981, Roth 1982). In contrast, when agents on one side can match with multiple agents from the other side, there is no stable matching mechanism that is also incentive compatible for these agents (Roth 1985). Surprisingly, this non-existence result can be obtained via a counter-example in which some of these agents can manipulate a stable matching mechanism via "demand reduction" by dropping some of the higher ranked agents from their preference lists. In the rest of the text, we will focus on the deferred acceptance algorithm in which proposals are made by agents who can match with at most one partner.

Most of the other results generalize to some extent to a variety of models in which agents on one side can make multiple matches or matching involves monetary transfers as well. A comprehensive survey of the matching theory can be found in Roth and Sotomayor (1990).

## *3.2 One-sided matching*

When one side consists of objects to be consumed in the model above, the model becomes a one-sided matching model. The simplest of these is the housing market model introduced by Shapley and Scarf (1974), in which one side of the market consists of agents each of whom own a single housing unit, and the other side consists of houses to be exchanged among the owners. Shapley and Scarf attributed the following exchange algorithm to Gale in their paper and named it the *top trading cycles* algorithm:

> Each agent points to the owner of his most preferred house. By finiteness, there exists a cycle of agents pointing to one another. Each agent is assigned the house of the owner that he points to in a cycle. The algorithm is repeated with the remaining owners and houses.

The top trading cycles algorithm yields an allocation that is in the core of the economy (Shapley and Scarf 1974) and therefore is Pareto efficient. When owners rank houses in strict preference order, the core is unique and coincides with the competitive equilibrium outcome (Roth and Postlewaite 1977). The direct mechanism defined by the top trading cycles algorithm is dominant strategy incentive compatible (Roth 1982). Furthermore, it is the only efficient and dominant strategy incentive compatible mechanism that assigns each owner a house that he weakly prefers to his own (Ma 1994).

This model can be seen as a variant of the two-sided model in which each house prefers its owner the most and stability is not a constraint in reallocation. When viewed this way, this model can be adopted for house allocation problems (Hylland and Zeckhauser 1977) by allowing house preferences over owners to be determined by a random rule (Abdulkadiroglu and Sonmez 1998), for house allocation problems in which some of the agents are existing tenants and house preferences over agents can be determined by a mix of random and nonrandom rules such as seniority  (Abdulkadiroglu and Sonmez 1999), and for models in which some objects have multiple copies (Abdulkadiroglu and Sonmez 2003). Complete characterizations of similar mechanisms can be found in Papai (1999) and Pycia and Unver (2016).

## 3.3 Real-world applications of matching mechanisms

These mechanisms and their variants have been put into use in various matching markets. In particular, the deferred acceptance algorithm was discovered independently by an entry-level labor market for medical interns in the early 1950s. Until then, the market had operated in a decentralized manner, culminating in congestion and offers to students made as early as two years before graduation. Various medical groups organized a centralized clearing house and invented a variant of Gale and Shapley’s deferred acceptance algorithm in which hospitals make the proposals (Roth 1984). Today known as the National Resident Matching Program (NRMP), the algorithm went through various changes since then, most notably in 1997 when the NRMP Board of Directors decided to switch to a student-proposing version following a controversy centering on employee welfare and incentives, which can be traced to the theoretical results mentioned above (Roth and Peranson 1999). The deferred acceptance algorithm operates in numerous entry-level markets for professionals today (Roth 2008). Variants of it have also been adopted in various college admissions markets (Balinki and Sonmez 1999).

One of most prominent newer applications of the deferred acceptance algorithm is found in public school choice, introduced by Abdulkadiroglu and Sonmez (2003). The practice of market design for school choice was initiated independently by the redesign of the student assignment systems in Boston and New York City. The former followed a 2003 Boston Globe article (Cook 2003) on Abdulkadiroglu and Sonmez (2003), which described incentive flaws with the student assignment mechanism used in Boston at the time. The latter was initiated independently when, after becoming aware of the pioneering market design work in entry-level labor markets (Roth, 1984; Roth and E. Peranson, 1999), the New York City Department of Education contacted Alvin E. Roth in 2003 to inquire about the possibility of adopting a system like the NRMP for their high school admissions to overcome the congestion problem that was being experienced in high school admissions.

The contrast between the two markets is informative. Boston Public Schools had used from late 1980s to 2005 a variant of the deferred acceptance algorithm, the so-called "immediate acceptance" algorithm, in which acceptances are final at every proposal stage. As documented in Abdulkadiroglu and Sonmez (2003), the immediate acceptance algorithm could be manipulated by savvy parents at the expense of others, a phenomenon that apparently had already been discovered by parents in affluent neighborhoods of the city (Abdulkadiroglu, Pathak, Roth and Sonmez 2005; Pathak and Sonmez 2008). After the aforementioned Boston Globe article brought the issue to public attention, BPS initiated the redesign efforts by making parental incentives a central issue. A task force appointed by BPS studied the stable matching mechanism of Gale and Shapley and an efficient matching mechanism inspired by Gale's top trading cycles algorithm (Abdulkadiroglu and Sonmez 2003). Initially the task force recommended the efficient matching algorithm; however the School Board voted for the deferred acceptance. One motivation for that choice was the potential to match students closer to home under deferred acceptance because of walk-zone priorities granted to students living within walking distance from schools, although simulations with field data revealed little difference between the mechanisms in terms of efficiency and distance between home and assigned school.

Despite its poor incentive properties, the old Boston admissions algorithm had been successful in clearing the market. By contrast, congestion was a major problem in New York City high school admissions. More than thirty percent of about 90,000 incoming high school students could not be matched to any of their choices by the end of March 2003 (Abdulkadiroglu, Pathak and Roth 2005). Unlike Boston, the New York City high school matching market was two-sided. A substantial number of New York City schools could rank students in preference order. Schools also self-reported available seats. It was reported that some schools were concealing their capacities in order to match with students of their choice outside the centralized matching mechanism. Despite that there is no matching mechanism that provides straightforward incentives for both parents and schools (Roth 1982), there is a sense in which a stable matching reduces the scope for manipulation by schools (Abdulkadiroglu, Pathak and Roth 2009).

Therefore, family incentives as well as school incentives, along with welfare concerns, motivated the choice of the deferred acceptance algorithm in New York City.

Gale and Shapley's deferred acceptance algorithm is at work in various other school districts today, including Chicago, Denver, and Washington DC. The San Francisco Unified School District was the first to adopt an admissions system based on top trading cycles in March 2010. The Recovery School District of New Orleans also adopted a version of top trading cycles for the first year of its centralized admissions, then switched to the deferred acceptance algorithm in order to meet legal regulations at some of the newly added schools to the admissions system in the second year.[21] Despite its poor incentive properties, variants of the Boston mechanism have also been adopted widely in high school and college admissions throughout the world (Chen and Kesten 2014).

The most prominent recent applications of exchange mechanisms inspired by top trading cycles are found in the kidney exchange literature (Roth, Sonmez and Unver 2004). The preferred treatment for end stage renal disease is kidney transplant. A significant supply of kidneys comes from live donation. However blood type or immunological incompatibilities between donor and intended recipient creates demand for kidney exchange. Kidney exchange was first suggested in 1986 by Felix T. Rapaport, a leading surgeon and scholar in the field of organ transplantation. The first exchange in the US was performed in 2000 (Wallis, Samy, Roth and Rees 2011). Despite the possibility of exchange among large groups of patient-donor pairs, legal and logistical concerns precluded exchange in large groups in the early years, yielding new theoretical research and subsequent design of exchange programs that focus on limited numbers of exchanges (Roth, Sonmez and Unver 2005, 2007). Today kidney exchange in the US has grown partly from the incorporation of nonsimultaneous exchange chains initiated by nondirected donors (Ashlaghi, Gilchrist, Roth and Rees 2011).

# 4 Common Themes

As we argued in the Introduction, our perspective in this survey is that auctions and matching are actually two sides of the same coin. Some general principles of market design transcend auctions and matching. For example, Roth (2008a) suggests that marketplaces need to provide thickness by attracting a sufficient proportion of potential market participants. Roth further argues that marketplaces need to overcome the congestion that thickness can bring and make it safe to participate in the market as simply as possible. We would add that, equally importantly, many of the practical challenges faced in auction and matching markets are similar in nature—and many of these common themes will, in turn, inevitably arise in future

[21] See Abdulkadiroglu (2013) and Pathak (2011) for surveys on school choice.

applications outside of auctions and matching. Before proceeding with our main point, the next section will attempt to convince the reader that there is a strong sense of continuity between the two sides of market design by showing examples from matching markets where nonmonetary instruments play the role of prices and examples from auction markets where the auctioneer deviates from using pure pricing mechanisms.

## 4.1 Cardinal vs. ordinal approach

Although the vast majority of the matching theory is built on the assumption of ordinal preferences, examples that incorporate cardinal elements in the final allocation of resources are prevalent in matching markets. For instance, in school choice, priority in assignment to students living within walking distance from a school is common practice. Such neighborhood priority along with housing markets creates an indirect pricing mechanism, offering wealthier or more motivated families the option of improving the likelihood of assignment of their children to schools of their choice by locating closer to these schools. A unique rule that tries to elicit cardinal preference information without resorting to direct or indirect pricing mechanisms comes in the form of the priority given to students whose families visit information sessions or open houses of schools in New York City. Similar examples of "burning money" appear in allocating free or reduced-price tickets to events, such as tickets for Blue Devils games; lines form and tents are assembled in front of Duke University's Cameron Stadium weeks before some of the games.

Recently, there has also been greater recognition of cardinal elements in matching theory. An extensively studied mechanism in the literature, known as random serial dictatorship (RSD) or the random priority rule, allows agents to pick their most preferred alternative among those available, one at a time, in a randomly determined order.[22] RSD and its variants are widely used in allocating campus housing and offices, as well as draft picks in professional sports. Bogomolnaia and Moulin (2001) showed that RSD can be improved upon unambiguously for any profile of cardinal preferences that are compatible with students' ordinal preferences. In particular, they show that this improvement can be obtained by increasing—in a first order stochastic dominance sense—every agent's likelihood of getting a higher choice without using any cardinal preference information. However, such improvement comes at the cost of incentives.

Likewise, Gale and Shapley's deferred acceptance algorithm is generally favored for school choice, as it is incentive compatible and its outcome is stable. However, it completely ignores underlying cardinal valuations. By contrast, the Boston immediate acceptance mechanism allows participants to express their

[22] When the random order in RSD is used as the school's preference at every school, the Gale and Shapley algorithm becomes equivalent to RSD.

intensity of preferences indirectly via their strategic behavior (Abdulkadiroglu, Che and Yasuda 2011). Although every complete-information Nash equilibrium outcome of the immediate acceptance mechanism is stable with respect to the underlying true ordinal preferences—and therefore dominated by the deferred acceptance outcome (Ergin and Sonmez 2006)—such strategic behavior may improve welfare. Suppose that a student's number one preference is for school A, which is routinely heavily over-subscribed, while the student's number two preference is for school B, which has less demand. In the student-proposing deferred acceptance algorithm, it is a dominant strategy to report school A ahead of school B, irrespective of the intensity of preference. However, in the immediate acceptance mechanism, if the student only mildly prefers school A to school B, it may be preferable for the student to report school B first; getting into school B for sure may be preferable to facing a lottery for school A and, if losing the lottery, facing the prospect that school B is also closed out. Resolution of this tradeoff in equilibrium may lead to more efficient allocation and Pareto improvement from an *ex ante* point of view. Abdulkadiroglu, Che and Yasuda (2014) show that, in large markets, gains of *ex ante* efficiency can be achieved via nonmonetary instruments. They propose a practical instrument, which allows students to signal some of their cardinal preference information and mimics the role of prices without forcing students to manipulate their ordinal preferences.

Such signaling mechanisms have evolved in various matching markets. A majority of top US colleges offer some form of early admissions through which a sizable number of students are accepted before the standard admissions date in the American college admissions market. Avery, Fairbanks and Zeckhauser (2003) report a 20 to 30 percentage point increase in acceptance probability associated with early admissions. Avery and Levin (2010) show how early admissions programs give students an opportunity to signal their enthusiasm. In a similar vein, the American Economic Association (AEA) also introduced a new mechanisms through which job market applicants can express interest ("signals") to up to two employers prior to interviews at the January meetings (Coles et al. 2010). (See also Section 5.6, below.) Lee and Niederle (2015) provide evidence based on a field experiment in an online dating market for how such signaling devices can increase the total number of matches.

Simultaneously, on the auction side of market design, there has been increased recognition of the limitations of a pure cardinal approach. One example of this has been the growing appreciation of the importance of budget constraints for bidders in auctions. Che and Gale (1998) observed that budget constraints may potentially reverse our standard prescription of second-price over first-price auctions. Since optimal bids in first-price auctions are shaded relative to full value, bidders are less likely to find their budgets to be binding. This upsets revenue equivalence and potentially results in first-price auctions producing higher revenues. Moreover, since bids are relatively more likely to reflect bidders' values than their limited budgets, first-price auctions may also yield more efficient outcomes than second-price auctions. Burkett (2014) demonstrates that conclusions may change qualitatively if budgets are allowed to be endogenous. The budget constraint may be a control mechanism imposed by a principal (e.g., the corporate board) on an agent (e.g., the manager delegated to bid) in order to curb managerial discretion. Therefore, a principal seeking to restrain an agent ought to set a relatively more stringent budget when the agent bids in a first-price rather than in a second-price auction, as identical budgets would leave the agent unconstrained in more states of the world in a first-price auction. This tends to negate any advantages of the first-price auction; and in some formulations of the model with endogenous budget constraints, full revenue neutrality is restored.

One place where there has been significant convergence between the auction and matching sides of market design is in combinatorial assignment problems such as course auctions. In traditional "bidding points auctions" used by a number of business schools, students are given fixed budgets of points and submit bids (in points) for the courses in which they want to enroll (Sonmez and Unver, 2010). Making side payments is not permitted, nor is purchasing additional points, so this is a mechanism squarely within the matching tradition. However, students can express their intensities of preference by concentrating their points on specific courses. Budish (2011) improves upon traditional bidding point auctions by treating students literally as agents in a classical exchange economy and by solving for an approximation to competitive equilibrium from equal incomes (CCEI). Approximate CCEIs are almost Pareto optimal and have other desirable properties.

## *4.2 Comparison of Mechanisms*

As Pathak (2016) argues for the design of public school choice programs, "what really matters are basic issues that market operators in other contexts would like to be concerned about." The theory helps to identify the tradeoffs that a designer may face and usually offers multiple solutions with varying degrees of resolution. However, the extent of these tradeoffs is of empirical concern and may vary from market to market. For example, despite the tradeoff between stability and efficiency, the deferred acceptance and top trading cycles algorithms cannot be theoretically ranked in relation to Pareto efficiency. Also the tradeoff may disappear under certain conditions (Ergin 2002, Ehlers and Erdil 2010). But simulations with field data reveal relatively sizable efficiency losses associated with deferred acceptance in the New York City High School Match.[23] In particular, Abdulkadiroglu, Pathak and Roth (2009) show that about 6% of students could be assigned a higher choice if inefficiencies in the deferred acceptance match could

[23] Interestingly, similar simulations with field data identify almost no efficiency loss in the Boston Public Schools match (Abdulkadiroglu, Pathak and Roth 2009).

be eliminated via Pareto improving swaps of school seats among students.[24] Yet, Abdulkadiroglu, Agarwal and Pathak (2015) show that the impact of such improvements on cardinal welfare is of second order relative to the gains from solving the aforementioned congestion problem in the New York City High School Match. They find that, when distance between home and school is used to quantify preferences, switching to the deferred acceptance algorithm realizes about 80% of the gains of a socially optimal benchmark. However, there are relatively modest gains from relaxing mechanism design constraints such as student incentives and stability (e.g. Erdil and Ergin 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Kesten 2010, Kesten and Kurino 2012). In particular, when there are ties in school preferences, no student optimal stable matching mechanism can be incentive compatible for students. Even if student preferences could be collected truthfully, use of student-optimal stable matching improves student welfare by a mere 0.58% on average. A matching mechanism that is Pareto efficient matching with respect to the ordinal preferences improves student welfare by another 3.3% of the range by abandoning the stability constraint.[25]

Just as the matching side of the literature has examined the comparison between the deferred acceptance and top trading cycles algorithms, the auction side has expended considerable effort on the comparison of different auction formats. There has been extensive theoretical, experimental and empirical work comparing: the first-price, second-price and English auctions, for a single item; and the pay-as-bid and uniform-price auctions, for multiple homogeneous goods. For example, the efficiency and revenue rankings of the pay-as-bid and uniform-price auctions are ambiguous (Ausubel et al. 2014). In practice, the choice of format often depends on which side of the Atlantic Ocean one stands; in both government securities auctions and day-ahead electricity auctions, the US tends to favor uniform pricing, while European countries often favor pay-as-bid pricing. As such, it is an interesting empirical exercise to compare the performances of the leading auction formats. Hortaçsu and McAdams (2010) do a structural estimation of bidding in pay-as-bid Treasury bill auctions in Turkey, and then perform counterfactual comparisons with uniform-price and Vickrey auctions. They find that the pay-as-bid auctions were very close to being fully efficient, and that switching to a uniform price or Vickrey auction would not have significantly increased revenues. Related work has been done in other government securities markets and in electricity markets. For example, Kastl (2011) estimates bounds on bidders' valuations from their bidding in the uniform-price Czech Treasury auctions. He concludes that the uniform-price auction performed well, in that it failed to extract at most 0.04% in annual yield of expected surplus while implementing an allocation that attained nearly the efficient surplus. Hortaçsu and Puller (2008) examined

[24] They also show that such improvements are incompatible with strategy proofness. Also see Erdil and Ergin (2008) and Kesten (2010) for earlier similar results.

[25] Also see Ehlers and Westkamp (2011) and Kesten and Unver (2015) for matching with ties in school preferences.

bidding in the Texas ERCOT day-ahead electricity market, also a uniform-price auction. They found that firms with large market shares performed close to the theoretical benchmark of static profit maximization, but that smaller firms deviated significantly from this benchmark, underlining the importance of payoff scale and learning in complex strategic market environments.

### 4.3 Caps and quotas

The previous sections have reviewed auction and matching mechanisms that, for example, determine efficient or stable allocations with respect to participants' expressed preferences. However, in many instances, a simple aggregation of individual preferences may run counter to societal objectives or ideals. For example, in school choice, a stable matching algorithm could result in segregated schools. In spectrum allocation, an efficient auction could result in a monopoly or duopoly for mobile voice and data services.

Allowing choice can potentially lead to more segregated schools. One can identify several causal channels. On the one hand, people may have a "taste for discrimination" (Becker 1957). On the other, segregation by race and class may increase as more savvy and motivated parents move their children to better schools, leaving many disadvantaged students behind in failing schools. Also, the dynamics in housing markets along with the common practice of neighborhood priority in admissions for students living nearby to schools may segregate neighborhoods along socioeconomic lines, inhibiting low income families' access to good schools. In all these cases, unrestricted aggregation of individual preferences by a stable or efficient matching mechanism does not necessarily meet societal objectives.

Policies targeting socioeconomic balance in school choice are implemented via quotas and set-asides. Quotas serve to limit the number of certain types of students to be admitted, set-asides target to achieve enrollment of a certain percentage of students of certain types. For example, in order to improve access to schools among students outside of the walk zone area of a school, Boston Public Schools used to grant neighborhood priority only for 50% of the seats at schools. In contrast, some schools in Denver Public Schools used to set aside a certain number of seats for students that qualify for free and reduced price lunch (FRL) and the schools can continue to seek FRL students if they cannot fill their set-asides during the regular admissions period.

Control via quotas is rather easy to accommodate in theory and in practice, especially when quotas are placed on non-overlapping categories (Abdulkadiroglu and Sonmez 2003). For instance, consider a school that grants priority for at most 60% of its seats to students applying from its feeder school, and priority for at most 60% of its seats to students coming from failing schools. When its feeder school is not classified as failing, the school can tentatively admits from its proposers according to its priority order subject to these quotas. That is, for example, if 60% of its seats are already assigned to students coming from its feeder, additional students coming from feeder schools are considered only for these seats and are not assigned to the remaining seats. This corresponds to the substitutes condition in school preferences (Abdulkadiroglu 2005), which is sufficient for the existence of stable matchings (Roth and Sotomayor 1990). Furthermore, the deferred acceptance algorithm is dominant strategy incentive compatible for families (Abdulkadiroglu 2005, Hatfield and Milgrom 2005). Such quotas may be imposed, for instance, on majority students in the expectation of improving the assignments of minority students. However, the impact of such quotas on student welfare is ambiguous. Kojima (2012) provides examples in which more stringent quotas for majority students may make minority students worse off and examples in which they may make all students weakly better off.

Set-asides, on the other hand, are seats reserved for specific types. For example, some schools in New Orleans may reserve certain percentages of seats for students coming from failing schools. Hafalir, Yenmez and Yildirim (2013) offer a stable solution in which set-asides are implemented via higher priorities for the associated types of students at these seats, otherwise available for all types of students. Echenique and Yenmez (2014) show that the deferred acceptance algorithm with higher priorities for the types eligible for set-asides is Pareto superior to the deferred acceptance algorithm with quotas. However, set-asides introduce complementarities among students when they are implemented as minimum quotas, which may preclude existence of stable matchings (Hatfield and Milgrom 2005). In fact, Ehlers, Hafalir, Yenmez and Yildirim (2014) show that, in that case, student priorities can be respected only among same-type students. They suggest a dynamic method to relax the set-aside constraints in order to achieve full stability, under which Gale and Shapley's student proposing mechanism continues to be dominant strategy incentive compatible and the school-proposing deferred acceptance algorithm minimizes violations of set-aside constraints.

Practical implementation of quotas and set-asides has implications for participants' welfare and incentives. For instance, half of the seats at Educational Option (EdOpt) schools in New York City are allocated via lottery, whereas the school can rank students in preference order for the remaining half (Abdulkadiroglu, Pathak and Roth 2009). When a student proposes to an EdOpt school within the deferred acceptance algorithm, he can be considered first for a random seat or for a preference seat. The order matters, since if a student highly ranked in the school's preference list also gets a good random number, he can be assigned a random seat, eliminating his competition for a preference seat, thereby increasing the chances for other ranked students to be enrolled at the school. After consulting with Abdulkadiroglu, Pathak and Roth, the New York City Department of Education decided to consider students for random seats first in order to increase the number of students assigned from schools' preference lists. In a similar vein, the Boston Public Schools used to give neighborhood priority for half of the seats at schools. In a recent work, Dur, Kominers, Pathak and Sonmez (2014) observe that whether a candidate is considered for a neighborhood seat first impacts the number of neighborhood students matched with schools.

In looking at telecommunications auctions, it should be recognized that spectrum is a scarce input into the provision of mobile voice and data services. Therefore, resolving the allocation strictly according to value maximization could result in monopoly, as that would potentially maximize the sum of bidders' profits in the downstream market. Conversely, a deliberate competition policy in the spectrum auction can facilitate new entry and improve the competitive performance of the downstream market.

Historically, the two most frequent devices for competition policy in spectrum auctions have been the set-aside and the spectrum cap. With a *set-aside*, one or more spectrum blocks are reserved for a particular class of bidders. For example, the set-aside may be reserved for new entrants, while incumbent firms are excluded from bidding on the set-aside. With a *spectrum cap*, a limit is placed on the quantity of spectrum that a bidder is permitted to acquire or hold. Spectrum caps are essentially a version of quotas in the matching environment, and they may apply to winnings within a given auction or they may apply cumulatively to specified existing holdings as well as to acquisitions within the given auction.

A third type of preference that is used in the US, and occasionally elsewhere, is the *bidding credit*. Auction participants that are small businesses (or, historically, minority- or women-owned businesses) may receive substantial percentage credits toward their bids. It is sometimes argued that bidding credits—or, for that matter, set-asides and caps—are giveaways that operate counter to the revenue objectives of auctions. However, in auctions where incumbents have market power, subsidizing designated bidders or reserving spectrum for entrants may create extra competition and induce the unsubsidized incumbents to bid more aggressively, potentially improving auction revenues. Similarly, offering a price preference to domestic suppliers can reduce a government's procurement costs if domestic firms have, on average, higher production costs than foreign firms (McAfee and McMillan 1988, 1989, Ayres and Cramton 1996).

Set-asides, spectrum caps and bidding credits are easily integrated into clock auction designs. A spectrum cap is a simple limit on the quantity that each participant can bid for and win. A set-aside can be implemented by using two price "clocks": a price for reserved spectrum and a price for unreserved spectrum. Both prices stop increasing when the demand for all spectrum (reserved and unreserved) does not exceed the supply. The price for reserved spectrum stops increasing sooner if the demand for reserved spectrum does not exceed the supply. Note that a set-aside can be viewed as essentially equivalent to an "aggregate spectrum cap", which limits the quantity that the entire set of incumbent firms can win. A bidding credit is merely a discount on the payments by winning designated bidders.

Caps are also commonplace in applications of auctions in other sectors. As noted above, the rules of the US Treasury auction limit each participant to bidding on 35% of the total supply. The EDF Generation Capacity Auctions for electricity, which were described above, imposed a 45% cap. Again, one of the motivations has been to limit post-auction exercises of market power.

## 4.4 Package preferences

Expression of package preferences has been a longstanding concern in market design. For example, the early research on airport slot auctions by Rassenti, Smith and Bulfin (1982) recognized that landing slots and takeoff slots are strong complements; consequently, they propose an auction design that allowed bidding for pairs of landing and takeoff slots.[26] Similarly, each side in the National Resident Matching Program may have significant package preferences. Two partners may identify themselves as a couple and submit choice lists of identical length. The two lists of a couple are treated as a unit in the sense that the couple prefers to be assigned the same ranked choices together in their rank order. Some second-year residency positions require specific first-year positions as prerequisites, creating strong complementarities between the positions for interns. Meanwhile, some hospitals' programs wish to fill only an even number of positions if they cannot fill all of their positions, a complementarity in hospitals' preferences (Roth and Peranson, 1999). This last scenario uncannily parallels recent 4G spectrum auctions, in which many bidders valued two 5 MHz blocks of low-frequency spectrum at more than twice the value of a single block, creating aggregate demand curves focused on even numbers of spectrum blocks.

An extremely useful meta-theorem, applicable both to auctions and matching, is that standard mechanisms incorporating package bids perform well in environments where all items are substitutes, but may break down when the substitutes condition is violated. On the auction side, Ausubel and Milgrom (2002) prove that substitutes is a robust condition implying that the Vickrey-Clarke-Groves (VCG) payoffs lie in the core. More precisely, if all bidders have substitutes preferences, then the VCG outcome is the unique bidder-optimal point in the core; conversely, given any bidder who violates the substitutes condition, one can always construct preferences for hypothetical opponents, each with linear preferences, such that the VCG outcome will lie outside the core. On the matching side, Roth and Sotomayor (1990) show that substitutable preferences are sufficient for the existence of stable matchings in the many-to-one

[26] See Schummer and Vohra (2013) for a clever theoretical restriction on package preferences that allows analytical tractability.

model.[27] Hatfield and Milgrom (2005) provide a unifying substitutes condition for matching with contracts that reduces to standard notions of substitutability both in classical price theory and in matching without prices. Hatfield and Kojima (2010) define a unilateral substitutes condition that further unifies the theory by recovering several desirable theoretical results in matching with contracts that include explicit prices. Namely, it guarantees that the same agents are matched across all stable matches (rural hospital theorem, Roth 1986), that the deferred acceptance mechanism is dominant strategy incentive compatible for the proposing agents (when each can be matched to at most agent on the other side of the market; Dubins and Freedman 1981, Roth 1982), and that there is no other individually-rational matching[28] that all of the proposing agents would strictly prefer to the Gale-Shapley outcome (Roth 1982).

The presence of couples introduces complementarities that may preclude the existence of stable matchings (Roth 1984). Moreover, the problem of finding a stable matching becomes NP-complete (Ron 1990). Also, even when one exists, there may not exist a stable matching that is optimal for one side of the market, and stable matchings may fill different number of slots (Aldershof and Carducci 1996). Furthermore, the NRMP algorithm, a variant of the deferred acceptance algorithm, may fail to find a stable matching when one exists (Klaus, Klijn and Massó 2007; Perrault, Drummond and Bacchus 2015). Klaus and Klijn (2005) provide a sufficient condition on couples' preferences for the existence of stable matchings. Nguyen and Vohra (2014), on the other hand, show that every such problem with couples has a nearby instance in which hospital capacities are perturbed slightly and the nearby instance has a stable matching.[29]

The holy grail in the auctions area has been a mechanism that reasonably accommodates environments rich with complements and substitutes, while retaining some degree of simplicity and transparency. While preferences for some commonly-auctioned items—such as Treasury securities and energy products—are reasonably consistent with substitutes, other important classes of items exhibit complementarities. For example, in spectrum, licenses for adjacent geographic regions are generally regarded as complements (Ausubel, Cramton, McAfee and McMillan, 1997), while licenses for different spectrum bands may be substitutes or complements. The VCG mechanism (Vickrey 1961, Clarke 1971, Groves 1973) makes truthful bidding a dominant strategy, but requires bidders to report $2^N$ values when there are $N$ items. Moreover, when the items are complements, revenues may be uncompetitively low and non-monotonic,

[27] Hospital preferences over interns are said to be substitutable if, when a hospital chooses an intern from a set of interns, the hospital chooses the same intern from a smaller subset as well. This definition rules out potential complementarities among interns, such as complementarity between a surgeon and anesthesiologist.

[28] A matching is said to be individually rational if no agent unilaterally prefers to break a match with a partner.

[29] Such a nearby instance perturbs each hospital's capacity by not more than four slots and increases the total number of slots across all hospitals by not more than nine slots.

and the mechanism may be vulnerable to loser collusion and shill bidding (Ausubel and Milgrom 2002, 2006; Yokoo et al. 2004). The most suitable mechanism developed to date is probably the combinatorial clock auction (CCA). The CCA includes a two-stage bidding process, consisting of a dynamic auction stage in which bidding occurs as in a standard clock auction, followed by an additional sealed-bid stage ("supplementary round") in which bidders can improve their bids from the first stage and submit additional bids as desired for other combinations. Throughout both stages, all bids are treated as all-or-nothing package bids. Winners are determined according to an optimization among all feasible combinations of bids, while pricing is based upon the notion of a core-selecting mechanism. The CCA was proposed by Ausubel, Cramton and Milgrom (2006). From 2012 to this writing in 2016, the CCA has been used for more than 10 major spectrum auctions worldwide, allocating prime low-frequency spectrum on three continents and raising approximately $20 billion in revenues (Ausubel and Baranov, 2016a). Cramton (2013) outlines the flaws present in designs without package bidding and argues how the CCA design solves them. Ausubel and Baranov (2014) summarize the recent history of the CCA, including expected innovations. Pricing issues and the pricing mechanism used in CCAs have been studied by Parkes (2001), Ausubel and Milgrom (2002), Day and Raghavan (2007), Day and Milgrom (2008), and Day and Cramton (2012).

## 4.5 Compact elicitation of preferences

Another recurring common theme, sometimes intertwined with the expression of package preferences, has been the compact elicitation of preferences. An illuminating case study is the design of sponsored search auctions, as are used for internet search engines such as Google, Bing and Yahoo. These auctions, in which advertisers bid for search page slots corresponding to various keywords, have attracted a great deal of attention. They also probably generate the majority of Google's substantial revenues. They are best known for utilizing the "Generalized Second-Price" (GSP) mechanism: the highest bidder wins the first position on the page but pays the second highest bidder's price, the second highest bidder wins the second position but pays the third highest bidder's price, etc. While the GSP mechanism "emerged in the wild," market designers have shown that it has a continuum of Nash equilibria in pure strategies and that the locally envy-free equilibrium (a refinement related to stability in matching) in the lowest prices is especially attractive (Edelman, Ostrovsky and Schwarz, 2007; Varian, 2007). In particular, it gives the same outcome as the unique Bayesian Nash equilibrium of an ascending clock auction corresponding to GSP and bidders receive the same payoffs as they get in the Vickrey-Clarke-Groves mechanism.

One underappreciated aspect of the sponsored search auction design is that it elicits bidders' valuations in an extremely compact form. A standard VCG auction would have each advertiser submit one bid representing its value for receiving the first slot, a second bid representing its value for receiving the second slot, a third bid representing its value for receiving the third slot, etc. Instead, the sponsored search auctions cleverly express bid amounts in *dollars per click*, and then ask each advertiser merely for a single scalar number. Note that there is no loss of generality in this compact representation, provided that the value of position *i* to advertiser *k* equals $\alpha_i s_k$, where $\alpha_i$ (the click-through rate from position *i*) is the same for all advertisers and $s_k$ (the value per click of advertiser *k*) is the same for all positions. (Moreover, the use of this representation is not special to GSP; for example, one could employ the same compact representation if one instead decided to use VCG pricing.) This compact elicitation of preferences economizes on advertisers' bidding effort and its advantages could potentially outweigh its disadvantages even if advertisers' values do not perfectly satisfy the $\alpha_i s_k$ functional form.

A similar theme shows up in many other places in the auction literature. For example, Milgrom (2009) addresses the message-length problem by analyzing auctions and exchanges that restrict bidders' reports to take a particular compact form: messages describe preferences indirectly as the value of a linear program; and are limited to demanding only integer quantities of goods, with local rates of substitution between any pair of goods of zero or one. Milgrom's framework extends the Shapley and Shubik (1972) assignment mechanism in three main ways: it allows participants to trade multiple types of goods, to trade multiple units of each type of good, and to take the role of buyer in relation to some goods and seller in relation to others. The combinatorial clock auction (CCA), as described in Section 4.4, has generally taken package bids to be mutually exclusive. While such a bidding language allows the full expression of preferences, it potentially requires bidders to place as many as $2^n$ bids in auctions of *n* items. As a result, the Canadian 2500 MHz spectrum auction in 2015 (with 318 licenses offered) enriched the CCA design to permit bidders to submit non-mutually-exclusive "OR" bids in the supplementary round, enabling much more compact bid submissions (Ausubel and Baranov, 2016a). The related computer science literature approaches the topic of bidding languages in much greater generality. A good starting point for the reader is the excellent overview by Noam Nisan (2006).

Precisely the same issue of compact elicitation of preferences arises in the matching literature. As noted by Milgrom (2009), in the National Resident Matching Program (NRMP), a hospital that interviews 50 candidates in the hopes of employing 10 could potentially require an ordinal ranking of all $1.3 \times 10^{10}$ subsets of 10 or fewer doctors to fully express its preferences. The NRMP restricts the message space by limiting reports to be ordinal rankings of *individual* doctors and a maximum number of slots to be filled. Observe that this restriction has no loss of generality when hospitals have responsive preferences—essentially the ordinal counterpart to additively-separable utility functions—an assumption that is frequently made in the literature (Roth and Sotomayor, 1990). As above, this compact elicitation of preferences economizes on hospitals' effort and its advantages could potentially outweigh its disadvantages even if hospitals do not perfectly satisfy the assumption of responsive preferences.

Finally, as noted throughout this survey, the classical matching literature takes an extreme ordinal view, in which underlying cardinal preferences are mostly ignored. Although this view has mostly historical roots, there is also a sense in which humans have a hard time precisely expressing their cardinal preferences (Sandholm and Boutilier, 2006). By contrast, a growing number of papers study signaling mechanisms in matching through which participants may express information regarding their underlying cardinal preferences in a simple and compact format (Avery and Levin 2010, Coles et al. 2010, Abdulkadiroglu, Che and Yasuda 2014, Lee and Niederle 2015). Such signaling devices allow market participants to act on their cardinal preferences instead of formulating them.

## 4.6 Player participation

Charter schools in the US are publicly funded but privately managed alternatives to traditional public schools. Since the opening of the first charter school in Minnesota about two decades ago, charter schools have become one of the fastest growing sectors in the American education scene. For example, 7 out of 87 elementary schools, 16 out of 44 middle schools and 16 out of 54 high schools are listed as charter schools by Denver Public Schools' choice program in 2014. After Hurricane Katrina in 2005, the Recovery School District took control of almost all public schools in New Orleans and authorized charter schools across the city. The district has become the country's first all-charter as of May 2014.[30]

Laws across states typically provide charter schools with great flexibility in running their own admissions lotteries. Charter schools can set their own deadlines and, to some extent, their own admissions rules. As the number of charter schools in a district increases, decentralized charter school admissions poses a major challenge for parents attempting to navigate all of the options. Indeed, Denver Public Schools was the first district to adopt a centralized choice plan that included charter schools in 2011. Yet participation in the choice plan was voluntary for charter schools.

Empirical evidence (Roth and Xing 1994) and laboratory experiments (Kagel and Roth 2000) show that stability becomes a major determinant for the success of a mechanism when participation is voluntary. Consequently, following economists' recommendation, Denver Public Schools adopted a stable matching algorithm for their admissions. Furthermore, the district put significant effort into understanding and accommodating admissions policies of charter schools, which vary uncharacteristically across schools.

[30] See http://www.washingtonpost.com/local/education/in-new-orleans-traditional-public-schools-close-for-good/2014/05/28/ae4f5724-e5de-11e3-8f90-73e071f3d637_story.html

In contrast, having tighter control of its schools, the Recovery School District adopted a centralized efficient admissions system based on Gale's top trading cycles algorithm in 2012 to allocate almost all of the charter school seats.[31] The district switched to a stable matching mechanism in 2013, because of the involvement of the State scholarship programs in admissions and the requirements of Act 2 that regulates scholarship programs.

All New Orleans public schools are run by the Recovery School District and all new charter schools are required to join the centralized admissions process. However, several Orleans Parish School Board charters chose not to participate until their contracts came up for renewal. Also, although the Board had initially entered its five conventional schools into the centralized admissions process, it voted unanimously after the main round of the 2013-14 admissions to pull back their participation in the follow-up rounds, after which two Orleans Parish schools, among the most popular in New Orleans, admitted a significant number of students in the summer of 2013 despite that they had earlier declared their slots fully assigned.[32] Among the reasons for pulling out that was stated in a newspaper article was the potential failure of the system to accommodate school preferences:

> "When a computer picks your students, you may not end up with the football team you're used to, or enough trumpeters in the band."[33]

The roots of this problem trace back to a complementarity problem; see Section 4.4. The example also highlights accommodation of school preferences in matching mechanisms as a major policy instrument to secure participation by schools that are allowed to run their own admissions. This school choice example resembles the exposure problem in auctions. For example, spectrum licenses may be offered at the regional level, and different regions may be complementary. As a result, the value that a wireless operator obtains from the combination of a New York license and a Washington DC license may be greater than the sum of their stand-alone values. When the auction format does not permit package bids, a bidder who bids above the stand-alone value of either license is exposed to the risk of winning only that license. This is known as the "exposure problem"—it poses a deterrent to bidders' participation in the auction, as well as a deterrent to bidding aggressively up to their full values.

The issue of player participation emerges in kidney exchange markets as well. Initial work on kidney exchange focused on incentives for patients and their surgeons (Ashlagi and Roth 2012) and early applications focused on exchanges in single hospitals or closely connected networks (Roth, Sonmez and

[31] See http://www.nola.com/education/index.ssf/2012/04/centralized_enrollment_in_reco.html

[32] See http://www.nola.com/education/index.ssf/2013/11/two_new_orleans_high_schools_s.html

[33] http://www.nola.com/education/index.ssf/2013/06/orleans_parish_school_board_pu.html

Unver 2005). With the formation of bigger exchange networks, hospitals have emerged as major players and hospitals' voluntary participation in exchanges has become a broader problem today (Ashlagi and Roth 2014).

Unlike complementarities in package auctions or school choice, the problem in kidney exchange is rooted in individual rationality. Namely, when a hospital can match a kidney in-house and the centralized clearing house does not pay attention to whether this transplant can be done internally, the hospital may prefer to conceal it from the clearing house. Ashlagi and Roth (2014) offer the first systematic study of the problem along with a new mechanism that gives hospitals incentives to reveal all patient-donor pairs.

Player participation has also been a recurring theme in the auctions literature.[34] For example, in symmetric single-item models, it has been argued that attracting one more bidder to participate in an auction has greater impact on the seller's expected revenue than reserve price policy or any other structuring of the negotiation process (Bulow and Klemperer 1996). In the context of the FCC's Incentive Auction, to be discussed in Section 5.5, one of the key rationales for the descending-clock reverse auction design is that its simplicity and strategy-proofness are important for reducing participation costs, especially for small local broadcasters whose participation is needed for a successful incentive auction (Milgrom, Ausubel, Levin and Segal 2012; Milgrom and Segal 2015).

## *4.7 Commitment (binding bids)*

Markets or mechanisms may fail when they do not provide participants with incentives to commit to their promises. Commitment issues arise in various forms in matching markets. Presumably the most prominent example comes from kidney exchange. For ethical reasons, donors must have the opportunity to withdraw their consent at any time (Ross et al. 1997). Since it is also difficult to write legally binding contracts about the future provision of a live donor kidney, a non-simultaneous exchange carries the risk of a donor withdrawing her consent after her intended recipient receives a transplanted kidney (Roth, Sonmez and Unver 2005). Therefore it is presumed that only simultaneous transplantation can ensure that recipients in an exchange receive their grafts (Ross et al. 1997). Since each patient and each donor require their own operating room and operating team in a simultaneous operation, this imposes a limit on the number of exchanges performed simultaneously. Therefore early applications focused on pairwise exchanges between two patient–donor pairs. Building upon Bogomolnaia and Moulin (2004), Roth, Sonmez and Unver (2005) characterize a class of efficient and incentive compatible pairwise exchange mechanisms under the assumption that recipients are indifferent among all compatible donors, a version of which was adopted by The New England Program for Kidney Exchange (NEPKE).

[34] Early contributions included Vickrey (1961) and McAfee and McMillan (1987b).

Exchanges among larger groups of pairs are performed around the world today. For example, NEPKE started implementing an algorithm that allows for four-way exchanges following Roth, Sonmez, and Unver (2007), who show that two- and three-way exchanges capture almost all of the gains from exchange, and full efficiency can be achieved by inclusion of four-way exchanges in large exchange programs (Sonmez and Unver 2013).

However, as kidney exchange has grown in practice, long chains initiated by non-directed altruistic donors have proven effective. Ashlagi, Gamarnik, Rees and Roth (2012) show that allowing long chains significantly increase efficiency in populations with high percentage of highly sensitized patients for whom a suitable kidney is difficult to find even from same blood type donors because of tissue incompatibility. Given the risk of reneging in nonsimultaneous chains, whether such chains should be implemented simultaneously or nonsimultaneously is at the center of a recent debate (Ashlagi, Gilchrist, Roth, Rees 2011). As we discussed above, higher percentage of highly sensitized patients in the exchange pool in comparison to the general population of patients is partly due to hospital incentives to conduct their own exchanges for easy-to-match pairs. Therefore mechanism that provide hospitals with incentive to fully participate in exchange are likely to reduce the need for long chains (Ashlagi and Roth 2013; Ashlagi, Fischer, Kash and Procaccia 2012). Alternatively Ausubel and Morrill (2013) recommend sequencing operations such that a recipient receives her graft only after the associated donor donates his kidney, eliminating the risk of reneging and allowing for implementation of long chains and cycles via a sequence of only two operations at a time.

The common problem in organ transplantation is the shortage of human organs. Introduction of monetary incentives in the market for live and cadaveric organ donations can potentially increase the supply of organs for transplant sufficiently to eliminate the very large queues in organ markets (Becker and Elias 2007), but such transactions are strictly prohibited in most countries (Roth 2007). On the other hand, non-monetary incentives for deceased donation have become a promising approach to increase supply of deceased organs. For instance, an individual who has previously registered as a donor gains priority on organ donor waiting list in case he becomes an organ transplant patient in Israel and Singapore. Such allocation schemes are proven to work in laboratory experiments when everyone who registered as an organ donor would actually donate (Kessler and Roth 2012). However, Israeli donor registration cards allow the donor to require a clergyman chosen by the deceased's family to approve the donation at the time of death. This creates a loophole for individuals to receive priority even though they would never make a donation, expecting their family or clergyman to decline the donation if and when in a position to do so (Kessler and Roth 2014a).

Enrollment planning is a major challenge for school districts. Even when a school accepts applicants up to capacity, natural reasons such as moving out of the district may cause some families not to register at the school. However, a centralized choice system makes it easier to apply to schools on one single application form, potentially allowing parents to list schools with which they are not familiar—potentially leading them not to show up at the school when they are accepted. When some of the schools pulled out of the centralized admissions in 2013 in New Orleans, one of the major criticisms was that the centralized admissions did not require parents to visit schools before applying:

> "They have no insight on what the school is capable of doing to their children … ." As for the students who applied through OneApp, "We don't know if they'll be able to benefit our school or if they'll lower our standards … ." Assigned school spots were vacant because administrators didn't know if children would show up.[35]

This was potentially a matter of greater concern in New York City in the early 2000s, when the city high schools went through a transformational reform. The city closed many failing large high schools, replacing them with more than 200 small schools, most of which offered theme-based curricula, such as information technology, math/science, arts, business, law, and environment (Abdulkadiroglu, Hu and Pathak 2014). These schools were new for families, with no historic record, most of them with catchy names and some with unfamiliar themes. Disseminating information about the schools has been a major goal. To this end, high schools hold information fairs throughout the city during the registration period. Families who visit a school or its stand at one of these fairs gain priority in admissions to the school.

Commitment is even a concern in entry-level labor markets such as the NRMP. Indeed, it is the lack of commitment which gives rise to the emphasis on stability in the first place. In many legal regimes, courts are unwilling to enforce labor contracts against workers, as doing so would condone indentured servitude or slavery. For example, in the United States, such enforcement is deemed to be contrary to the Thirteenth Amendment of the US Constitution.

Ensuring that bids are binding is also a first-order concern in the auction side of market design. However, unlike in matching markets, since money changes hands, there frequently exist easy ways to enforce commitments. The classic example exhibiting the potential difficulties in an auction with non-binding bids was a sealed-bid first-price auction for two Australian satellite television licenses in 1993. As reviewed by McMillan (1994), the winning bidders submitted "startlingly high bids" but "had no intention of paying their bids; they had bid high merely to guarantee they won." Rather, they had also submitted a series of lower bids and defaulted in sequence on each successive bid until reaching the

[35] See http://www.nola.com/education/index.ssf/2013/06/orleans_parish_school_board_pu.html

lowest bid that still exceeded all opponents' prices. With this fiasco in mind, most auctions today require participants to post financial guarantees (such as deposits or letters of credit) in order to bid, and default—meaning the failure of a winning bidder to consummate the transaction—results in forfeiture of all or part of the financial guarantee. The exception that proves the rule is diamond auctions, in which financial guarantees are generally not required. Diamond traders tend to belong to one of a small number of trading networks; reneging on even a single commitment is likely to result in exclusion from future trading.

The most notorious recent example of an auction with non-binding bids is Medicare's auction for durable medical equipment. In this procurement auction, winners are paid the median winning bid. The auction may leave demand unfilled, as some winners may refuse to supply (without penalty) because the price is set below their cost (Cramton, Ellermeyer and Katzman, 2015). At the same time, the lack of commitment invites the use of lowball bidding strategies, employed empirically and replicated experimentally. As a result, the Medicare format is outperformed by a conventional uniform-price auction in laboratory experiments (Merlob, Plott and Zhang, 2012).

## 4.8 Timing (offers too early or too late)

One common concern in market design is whether the timing of offers is conducive to efficient trade. However, there is an instructive contrast between the matching and auction sides of market design on this issue, and the concerns frequently go in opposite directions.

On the matching side, problems related to the timing of offers tend to be caused by the incentives that sometimes lead employers to try to match slightly earlier than their competitors (Roth and Xing 1994), leading to employment offers being made very far in advance of the relevant start of work, or employment offers with very short fuses ("exploding offers"). For instance, in the classic example of the entry-level labor market for American doctors (Roth 1984), by 1944, employment contracts were being arranged between hospitals and medical students two years in advance of graduation. Such "unravelling" occurs both in markets in which prices can adjust freely, e.g. new associates in law firms, and in markets in which they cannot, e.g. judicial clerkships in federal courts. Early offers in matching markets constitute a major inefficiency due to costs of potential mismatches caused by uncertainty of employees' qualifications before the completion of their training, and hiring in anticipation of uncertain future needs (Roth and Xing 1994).

By way of contrast, in dynamic auctions, it is taken to be a sign of trouble when bids are predominantly being made near the very end of the auction. For example, a significant fraction of bids in eBay auctions are submitted very near the fixed ending time, a practice known as "bid sniping". Note that the prevalence of this phenomenon runs cross-purposes to operating a dynamic auction—and may be said effectively to convert the dynamic auction into a sealed-bid auction.

One of the lasting innovations of market design for the initial FCC spectrum auctions was the introduction of activity rules (Milgrom 2004). A bidder is required to submit bids in early rounds of the auction in order to retain the right to submit bids for like-sized quantities in later rounds. For example, every license is assigned a specified number of points, and the total points associated with all of the licenses in a bidder's bid is not allowed to increase from round to round. This has the effect of sharply restraining the occurrence of bid sniping. Virtually all spectrum auctions worldwide in the past two decades have been dynamic auctions with activity rules.

Unfortunately, activity rules based purely on monotonicity in points run into the following difficulty: for any choice of points, there exist bidder valuations and potential price histories such that straightforward bidding is not possible under the activity rule. For this reason, improvements in activity rules have been a fertile area of research. Ausubel, Cramton and Milgrom (2006) introduced the notion of activity rules based upon the Weak Axiom of Revealed Preference (WARP). Many of the combinatorial clock auctions conducted in recent years have included activity rules based on WARP. Ausubel and Baranov (2014, 2016b) argued recently that a strict application of the Generalized Axiom of Revealed Preference (GARP) is the most natural activity rule. Each bid is taken to be a price-quantity pair; a new bid is deemed to satisfy the activity rule if and only if it, together with all prior bids, remains a data set consistent with GARP. This is the strictest activity rule under which straightforward bidding is always allowed, yet it satisfies the "no dead end" property that, after any bidding history, one of the bidder's previous bids (and all smaller sets of items) is always allowed.

## 4.9 Market Size

Enlarging the size of the market will often improve its performance, for a variety of reasons. First, it may reduce the market power of individuals and diminish informational rents. Second, it may reduce the complexity of trades, for example by shortening trading cycles. Third, it may increase the likelihood of stable matches, notwithstanding features that prevent any general guarantee of existence.

In the auction context, attracting one more bidder will often have greater impact than the choice of auction mechanism or making clever use of bargaining. With symmetric bidders and independent values, a standard auction (e.g., first-price sealed-bid or English auction) with no reserve price for $N + 1$ bidders will raise higher expected revenues than the optimal auction for $N$ bidders (Bulow and Klemperer, 1996). Similar considerations hold in the matching context, where increasing the number of potential match partners may also produce benefits by reducing market power.

Recall the well-known tradeoff between incentives and stability. As reviewed in Section 3.1, there is no stable mechanism that is strategy-proof for both sides of a matching market (e.g., men and women in the marriage model) or for agents that can match with multiple partners (e.g., hospitals in the NRMP). However, reporting true preferences becomes an approximate Bayesian equilibrium for hospitals in the resident-optimal stable match, as the hospitals' market power diminishes in large markets (Kojima and Pathak 2009).

A similar tradeoff exists between incentives and efficiency when symmetric treatment of similar agents is required. In particular, no symmetric mechanism can be both strategy-proof and ex ante Pareto efficient (Bogomolnaia and Moulin 2001). For instance, various school choice programs assign students to their most preferred available choice in a randomly determined order. This straightforward mechanism, referred as the random priority or random serial dictatorship in the literature, is strategy proof, but it is only ex post Pareto efficient. It can be improved in the sense of first-order stochastic dominance by assigning some students higher in their choice lists with higher probability without harming other students' allocations (Bogomolnaia and Moulin 2001). This trade-off vanishes in large markets as potential for such stochastic improvements diminish (Che and Kojima 2010).

Market power also relates to the size of stable matchings. Under fairly general conditions, the set of stable matchings forms a lattice (Echenique and Oviedo 2006), indicating that what is good for hospitals among stable matchings is bad for residents. The set can also be arbitrarily large in finite economies. However, Roth and Peranson (1999) observe that the set of stable matchings has in practice been small in the NRMP, limiting the extent of conflict between the opposite sides of the market. The authors argue that short preference lists submitted by the applicants in relatively large markets shrink the set of stable matchings. Immorlica and Mahdian (2005) offer a theoretical treatment of this argument. In contrast, Azevedo and Leshno (2012) give general conditions under which a model with finitely many hospitals and continuum of residents admits a unique stable matching.

Larger participant pools may also diminish the need for elaborate designs. This is particularly important in the context of donor organ exchange. As discussed above, because of enforceability issues, the extent of kidney exchange is limited by the availability of simultaneous operating teams and rooms at hospitals, forcing two-way exchanges among patient-donor pairs and theoretically precluding efficient exchange. However, computational results of Saidman et al. (2006) on real and simulated patient data show that, as the available population of incompatible patient-donor pairs grows, an increasing percentage of patients are able to receive transplants via two-way exchanges and most gains from trade can be captured with three-way exchanges.

Theoretical challenges may also diminish when the number of participants in matching markets is increased. Nonexistence of stable matchings becomes a theoretical possibility in the presence of couples. Nonetheless, couples in the NRMP have enjoyed match rates over 90% every year since 1984.[36] Moreover, it has been reported that the NRMP algorithm has succeeded in finding a stable match almost every year (Ashlagi, Braverman, and Hassidim (2014) and Kojima, Pathak and Roth (2013)). These authors explain this empirical puzzle by showing that the probability of existence of stable matchings converges to one in large markets if the market size grows sufficiently faster than the percentage of couples. By contrast, Che, Kim and Kojima (2015) study complementarities in hospital preferences over residents, a model that can also be applied in the context of school choice when schools care about the composition of the entering class regarding, for example, the school band or football team. Under a mild continuity condition, they show existence of stable matchings in large markets with complementarities.

# 5 New Frontiers of Market Design

As theory meets practice in market design, new questions driven by policy or practical considerations lead to new promising directions in research. Although open questions abound, we will focus on only a few broad themes that in our view pose new opportunities. Product design—i.e., defining the products that will be allocated in a matching or auction mechanism—is a neglected but critical aspect of market design. "Behavioral" market design, which includes deciding on the choices presented to market participants and on the default options, is a fertile area for progress. The design and improvement of secondary markets is a less studied topic with significant potential for research and impact. The US Federal Communications Commission's incentive auction program is quite an ambitious market design project with new sets of problems and challenges. Finally, the market design of the economics profession is itself a productive realm for innovation. We explore these new frontiers briefly in this section.

## *5.1 Product design*

The items that are allocated in an auction or matching mechanism are themselves "products" that can be designed. For example, the matching literature has so far focused on the design of student assignment mechanisms in school choice. Yet the design of the school portfolio offered within the school choice program is probably more important from a policy viewpoint. Demand data generated by an incentive compatible assignment mechanism may inform the decisions to replicate or close certain programs. More importantly, structural econometric models of demand for schooling can reveal the parental tradeoffs, consequently guide the design of new schools in a choice program. Recent research indicates that,

[36] See http://www.nrmp.org/wp-content/uploads/2014/04/Main-Match-Results-and-Data-2014.pdf

everything else being equal, parents prefer schools closer to their home; however they trade off distance for other school attributes such as academic achievement, school size and curriculum type (Hastings et. al. 2009; Abdulkadiroglu, Agarwal and Pathak 2014). Tastes for attributes vary across subpopulations. For example, although academically more successful schools are ranked higher across all populations, parents of students with higher baseline math test scores value academic achievement more in the New York City high school choice program. Such preference heterogeneity can be exploited in policy implementation. For instance, certain additional attributes, such as an after-school program, may benefit low-income neighborhood schools more than high-income neighborhood schools in attracting students to their math and science programs. This offers a unique opportunity to reconsider the market design for school choice: Designing school portfolios as well as assignment mechanisms by bringing matching theory and structural IO together.

Product design is also a critical aspect in the practical implementation of auctions, but it has tended largely to be ignored in the auctions literature.[37] In practice, a creative product definition can often reduce the complexities required by the auction design. For example, as mentioned above, early research into airport slot auctions by Rassenti, Smith and Bulfin (1982) emphasized the strong complementarity between landing and takeoff slots. In the plan published by the US Federal Aviation Administration (FAA) for 2009 slot auctions at the three New York City airports, the products to be offered were pairs comprising a landing slot and a later takeoff slot from the same airport. This product design does away with the extreme complementarity. At the same time, it still leaves a substantial need for package bidding, as there are significant complementarities for a concentration of slots in the same time interval, arising from the hub-and-spoke operations typically used by airlines.

Most auctions have offered predetermined products but a few auction designs have attempted to make the product design endogenous within the auction and "let the market decide" what products will be sold. For example, in early spectrum auctions, some specific frequencies were offered in 2x15 MHz blocks while other specific frequencies were offered in 2x5 MHz blocks, as regulators tried to anticipate the needs of bidders. However, in recent years, this has largely given way to offering "generic" 2x5 MHz blocks. In the main phase of the auction, bidders bid for and win any allowable number of generic blocks. Then, in a subsequent assignment stage, bidders' winnings are mapped to contiguous blocks of specific frequencies (Cramton 2013, Ausubel and Baranov 2014). In an early design for the UK auction of 2.6 GHz spectrum, the allocation of frequencies between "paired" and "unpaired" spectrum was also made endogenous. Bidders could submit package bids for arbitrary numbers of paired and unpaired blocks, and a band plan would be selected that maximized value as reflected in the bids (Cramton 2013). More broadly, many

[37] Nice discussions emphasizing aspects of product design are included in Milgrom (2004) and Cramton (2013).

combinatorial auctions can be viewed as attempts to make the product design endogenous, by dividing the items being auctioned into small pieces and allowing bidders to assemble their desired packages.

## 5.2 Research design and market design

Structural econometric techniques leverage predictive power by exploiting restrictions on data imposed by econometric theory and furthermore allow the study of counterfactual scenarios that are not readily available in the data. An influential and large structural econometric research program on auction markets has emerged over the last two decades. A growing literature also employs structural methods to study matching markets. We would not do justice in any attempt to survey these literatures here. Therefore, among many excellent surveys, we refer the reader to Paarsch and Hong (2006) and Athey and Haile (2007) for the econometrics of auctions[38] and to Chiappori and Salanie (2015) for the econometrics of matching.[39]

Here, however, we would like to emphasize an unprecedented opportunity for program evaluation in education that is introduced by unified enrollment systems with rigorously designed assignment mechanisms in school choice programs throughout the world. In particular, priorities in admissions, such as priority to students within a certain proximity to a school, are common practice in most programs. Furthermore, most assignment mechanisms rely on lotteries to ration seats among students with equal priority. Consequently, such lotteries create quasi-experimental variation in assignments to schools. Likewise, test-based admissions also create such variation around admissions cutoffs of schools. A recent and growing literature exploits such quasi-experiments from unified enrollment systems.[40]

However, assignment mechanisms weave random assignment into a complex structure of information on student preferences and school priorities. Consequently, earlier research either studied a subset of assignment data in which such randomization is identified easily, or partially ignored the information structure and relied on relatively weak instruments. Recently, Abdulkadiroglu, Angrist, Narita, and Pathak (2015) develop propensity-score based techniques to fully exploit quasi-experimental variation in the broad class of mechanisms that assign students with the same preferences lists and priorities to

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[38] Among many notable studies in the empirical auction literature, see for example Hendricks and Porter (1988), Hendricks, Porter and Wilson (1994), Paarsch (1992), Laffont, Ossard and Vuong (1995), Baldwin, Marshall, and Jean-Francois (1997), Guerre, Perrigne and Vuong (2000), Haile (2001), Athey and Levin (2001), Haile and Tamer (2003), Asker (2010), Athey, Levin and Seira (2011) and Roberts and Sweeting (2013).

[39] For examples on empirical matching, among others see Baccara, Collard-Wexler, Felli, and Yariv (2014), He (2014), Abdulkadiroglu, Agarwal and Pathak (2015), Agarwal (2015), Agarwal and Diamond (2015), Agarwal and Somani (2015) and He, Fack and Grenet (2015).

[40] For example, Hastings et al. (2009), Deming (2011), Abdulkadiroglu, Hu and Pathak (2013), Bloom and Unterman (2014), Deming et al. (2014), Abdulkadiroglu, Pathak, and Walters (2015) exploit lotteries in assignment; and Ajayi (2014), Dobbie and Fryer (2014), Lucas and Mbiti (2014), Pop-Eleches and Urquiola (2013), Jackson (2010), and Hastings, Neilson, and Zimmerman (2013) exploit discontinuity design.

schools with equal probabilities; furthermore they offer easily-implemented empirical strategies to use data from a deferred acceptance match. These techniques can be effectively used for program evaluation in the best possible way.

Market design indeed opens a new avenue for research design in the economics of education. Further econometric theories and empirical strategies are in order. Of equal importance is the question of whether we will be able to also incorporate sound econometric tools into our designs that would help districts evaluate their schooling alternatives beyond simple descriptive statistics and free of selection bias.

### 5.3 Behavioral market design

The behavioral economics literature provides an ample catalogue of systematic behavioral deviations away from the standard assumption of full rationality in economics, findings of which have already started a "behavioral economic engineering" approach in the market place, combining theory and laboratory experiments (Bolton and Ockenfels 2012).

Behavioral market design offers interesting possibilities for improving matching markets. Consider, for example, kidney transplants. Whether transplants come from live donation or cadaveric donation, the kidneys are supplied by donors. Yet recent research suggests that donors fail to exhibit fully rational behavior. The default option for cadaveric organ donation registry varies across countries. Johnson and Goldstein (2004) report the impact of default options on consent rates. In an online experiment, they ask respondents whether they would be donors using one of three questions with varying defaults. In the opt-in condition, the default is not to be a donor; in an opt-out condition, the default is to be a donor; in both cases, a respondent can confirm or change his donor status. Finally, a third neutral condition simply asks a respondent to make a choice between becoming a donor or not. They observe substantially higher donation rates under the opt-out and neutral conditions. Furthermore, they compare empirical consent rates among eleven European countries: the four countries with opt-in default policies have donation rates of only 4% to 27.5%, whereas among the seven countries with opt-out default policies, five achieve donation rates of 99.5% or higher (the other two having 98% and 85.9%).[41] However, presumed consent has also proved to be politically challenging or legally infeasible. Alternatively, a number of states in the US as well as UK have adopted an active choice, which asks individuals to select a "yes" or "no" for an organ donation request question. Kessler and Roth (2014b) report likely decreased registration rates in California after a switch to the organ donor registration question at the DMV from an opt-in frame to an active choice frame.

[41] However, among European countries that presume consent, only Spain has a higher per capita organ recovery rate than the US due to a more efficient organization of transplant services (Deffains and Ythier 2010, Kessler and Roth 2014a).

Roth (2007) also raises repugnance as a constraint on markets. In particular, because people have two kidneys and need only one to live, a market allowing the buying and selling of organs may greatly increase supply. However, a widespread repugnance for such transactions is reflected in laws against buying or selling kidneys. Although behavioral issues are not discussed as systematically in school choice, they arise in that context as well. One of the motivations for public school choice is to create competitive pressure for schools through parental demand. To this end and to help parents make informed decisions, districts provide ample information about schools regarding academic achievement, socioeconomic make-up, disciplinary punishment rates, etc. Yet how this information is presented may matter in parental decision making. For example, reporting rates of academic success versus rates of academic failure may shape preferences differently, resulting in different degree of competitive pressure for schools. One of the classic demonstrations of this phenomenon is provided in the context of treatment choice (McNeil, Pauker, Sox and Tversky 1982). When participants in a survey were asked to choose between surgery and radiotherapy for treatment of lung cancer, the choices were significantly different between the group that was presented with mortality rates for each procedure and the group that was provided the same information but presented as survival rates.

A second issue relates to the maximum number of schools that a family can rank in their application. For example New York City parents can rank at most 12 programs out of more than 600, forcing parents with more admissible choices to choose strategically and potentially harming efficiency and stability in the market (Haeringer and Klijn 2009). Yet, personal communication with New York City Department of Education officials also indicated a belief by policy makers that a limit on the number of choices may make the process of choosing from the many options less overwhelming for parents. A related issue is parental access to information. Although school districts spend considerable amount of resources in providing information about schools, information that is critical in most decision making is usually buried within parental networks. As noted by Jackson (2014) in a more general context, failing to include the impact of such networks on information diffusion can lead to inefficiencies and unfairness at the parental decision making stage and consequently at the matching stage.

Behavioral considerations in auctions are well recognized. Irrational overbidding is common in online auctions (Malmendier and Lee 2011). Although the Dutch auction and the first-price sealed-bid auction are strategically equivalent (Vickrey 1961), the Dutch auction generates lower (higher) seller revenue at faster (slower) clock speed in laboratory experiments (Katok and Kwasnica 2007). Some of these observations may be explained by the anticipation of regret by losing bidders (Filiz-Ozbay and Ozbay 2007). Variants of the clock auction may induce behavior counter to theoretical predictions, indicating that choosing between mechanisms based on their equilibrium properties may omit important and relevant considerations relevant to learning and lack of full rationality (Kagel and Levin 2009). Indeed, two of the commonly-cited advantages of dynamic auctions over sealed-bid auctions are difficult to understand in a fully-rational model: the potential for the "excitement" of a dynamic auction to generate higher prices; and, in multi-item settings, the potential for the iterative nature of a dynamic auction to allow bidders to economize on evaluating their values.

While behavioral economics has had relatively limited influence in market design to date, these examples all suggest that a research program of behavioral market design could be fruitful in the future.

## 5.4 Secondary markets

The preponderance of work to date on the auction side of market design has involved primary markets: a government or other large party is selling items; or, alternatively, a large buyer is procuring items. Comparatively less effort has gone into the design of secondary markets, i.e., resale markets of items between smaller sellers and buyers. A likely explanation for this stylized fact is that primary sellers and buyers have the most to gain from establishing an efficient or revenue-optimizing mechanism and, consequently, they have been the parties most likely to seek out market design expertise and to initiate market design projects. At the same time, one should never minimize the possibility that, when an existing trading mechanism is inefficient, some existing stakeholders may be beneficiaries of the existing system and they may actively oppose any redesign.

A good example of this occurs in secondary markets for equity. Budish, Cramton and Shim (2013) liken the competition by high frequency traders to shave milliseconds off communications times between the exchanges to an "arms race", with the twin costs of socially-wasteful expenditures and reduced market liquidity being borne by investors. They argue that the source of the problem is the continuous limit order book used on exchanges. As a solution, they propose replacing the continuous limit order book with uniform-price sealed-bid double auctions conducted at frequent but discrete time intervals.

The example of high frequency trading suggests that there may be a host of secondary markets today exhibiting substantial inefficiencies. Some of these markets contain subtle nuances that make solutions difficult even for the most experienced market designers. However, for other secondary markets, there are easily identifiable market designs that could readily be introduced, but such improvements are often blocked by vested interests.

Secondary markets occur in school choice as well. Parents' preferences may change after the primary assignment round due to change of home address, change of school location, inadequate provision of entitled services at assigned school etc. School districts allow unhappy parents to appeal their assignments. In 2004, the New York City Department of Education received just over 5,100 appeals, which constitutes about 5% of the main round assignments. Around 300 of these appeals were from students who received their first choice and about 2,600 of them were granted on a case-by-case basis (Abdulkadiroglu, Pathak, and Roth 2005). Given the lack of seat availability at the appeals stage, exchange of school seats among the appealing students provides a potential solution at the cost of harming stability and incentives in the main round. Designing an efficient appeals process remains a priority in school choice.

## 5.5 The FCC incentive auction

Probably the most ambitious market design project underway at this writing is the design and implementation of the US Federal Communications Commission (FCC) incentive auction. The premise for this project is that, in many countries, wireless data services have become a much more productive use of telecommunications spectrum than television broadcasting. In particular, the vast majority of American viewers receive their programming not via over-the-air signals, but by cable, satellite TV, or the Internet. Meanwhile, since the advent of smartphones, growth rates in wireless data usage have been exponential. However, television broadcast licenses are often tantamount to property rights; the spectrum becomes available only if broadcast licenses are relinquished voluntarily and, as a result, the incumbent licensees are in the position of sellers.

Thus, the FCC incentive auction poses notably thorny market design issues, in several respects. First, unlike the other spectrum auctions designed in the past two decades, it is a two-sided auction, with both buyers and sellers; the government stands largely as an intermediary and facilitator. Second, the "reverse auction" for incumbent television licensees involves an extremely complex graph of interferences, involving thousands of broadcast stations and hundreds of thousands of pairwise constraints, and where different stations may be substitutes or complements. Third, for UHF television licensees, there is not just one relinquishment option but three: moving to upper VHF frequencies, moving to lower VHF frequencies, or going off-air. Fourth, the "forward auction" for mobile telephone operators needs to run substantially faster than traditional US spectrum auctions, since it is part of a larger trading mechanism, and aspires to serve additional goals such as furthering competition policy in the mobile market, making spectrum available for unlicensed use, and raising funds for first-responder networks. Finally, the relevant stakeholders are more disparate than usual, including firms in two very different industries: broadcast media and mobile telephony. For discussions and proposed solutions, see Milgrom, Ausubel, Levin and Segal (2015) and Milgrom and Segal (2015).

Many of the proposed methods to address the incentive auction will seem familiar, given the toolbox described in Section 2. For the forward auction, a uniform-price ascending clock format is proposed. For the reverse auction, a descending clock format is proposed, in which the price stops descending for a given station when, in essence, it is guaranteed to be a winner in the auction. However, the extent of institutional and physical constraints, the routine violation of well-understood regularity conditions, the complexity of each component, the sheer number of moving parts, and the political interactions make this a thorny market design problem, indeed.

## 5.6 Market design of the economics profession

Many of the concerns that the emerging field of market design has directed outward to real-world markets could also profitably be directed inward. The economics profession itself has issues relating to the entry-level labor market, the admissions of new students, and the efficient generation, communication and certification of ideas. Economists may talk the talk of market design, but do they walk the walk?

The economics job market (at the Ph.D. level) today exhibits some of the characteristics advocated by the Gale/Shapley/Roth approach to medical and other professional markets. In particular, hiring activities are centralized to the extent that most screening interviews occur at the annual ASSA Meetings in January; however, activities after the screening interviews, especially the making of offers, are decentralized. The market is relatively thick, but competition for a limited number of interviews creates concerns of congestion. Both open-ended and exploding offers are common at the hiring stage.

At the suggestion of an Ad Hoc Committee (Coles et al. 2010), the American Economic Association (AEA) introduced two new mechanisms in 2006. First, job market applicants can express interest ("signals") to up to two employers prior to interviews at the January meetings. Second, the AEA employs a web-based scramble for candidates who are still on the market and for employers with vacant positions after the initial rounds of hiring. The first mechanism aims to solve coordination and congestion problems; the second aims to reduce search costs and thicken the latter part of the job market. The Ad Hoc Committee report (Coles et al. 2010) presents survey evidence of success in both dimensions.[42] Designing a market for job matching is presumably a more challenging task. Traditionally, negotiations on offer terms (e.g., salary, research support and teaching load) have been an integral part of the process. We predict that, unlike the NRMP, any centralized clearinghouse solution for economists would necessarily incorporate such offer terms, incorporating insights from the theory of matching with contracts and/or monetary transfers (e.g., Kelso and Crawford (1982), Hatfield and Milgrom (2005) and Hatfield and Kojima (2010)). Any successful application of a centralized clearinghouse in Economics would, in turn, be likely to impact the NRMP; there it has been argued that the setting of offer terms prior

[42] Coles et al (2010) argue that signals can serve as a coordination mechanism in breaking ties to fill scarce interview slots by placing more weight on candidates who signaled. Che and Koh (2015) offer new insight on how such coordination problems can be mitigated in decentralized admissions market as well.

to the match benefits hospitals by compressing wages for medical interns (Bulow and Levin 2006; also see Niederle 2007 and Agarwal 2015 for counter arguments).

The admissions process for graduate economics programs seems to display little in common with the idealized treatments of college admissions and school choice in the market design literature. In particular, departments make their admissions offers on a fully decentralized basis, and departments often scramble near the acceptance deadline to reallocate their assistantships to other prospective students as offers are declined. Moreover, financial support packages are an essential part of the admissions process, and graduate programs frequently apply very different admissions standards to funded versus unfunded students. As such, many of the comments about job matching made in the previous paragraph apply to graduate admissions as well. For that matter, it could also be observed that the American college admissions process today differs substantially from Gale and Shapley's idealized model of 1962—financial aid packages, which may vary on an individual basis, are important components of admissions offers.

Perhaps most fundamentally, there are a variety of ways in which the market for ideas in economics may perform suboptimally. There is some evidence that the peer review process[43] and promotion decisions[44] in academia may be clouded by aspects of an "old boy network." Conference programs in many fields of economics, perhaps even including market design, are sometimes viewed by outsiders as clubs. For as long as we can remember, the peer review process for publication has been widely accused of being inordinately slow while adding unnecessarily little in value.[45] Some interesting experiments have been attempted to improve both the quality and speed of the publication process. The Berkeley Economic Press introduced journal families with multiple quality tiers; the author could, with a single submission, have her manuscript considered for all of the quality tiers.[46] The *American Economic Review* for a time experimented with double-blind refereeing (Blank 1991), which became impractical with the rise of

[43] For example, a recent working paper (Colussi 2015) examines publications from 2000 – 2006 in the top four general interest journals in economics: the *American Economic Review*, the *Journal of Political Economy*, *Econometrica* and the *Quarterly Journal of Economics*. It observes: "ECA and the AER seem to be more open than the QJE and JPE, which show a bias towards authors appointed at their host institutions. Roughly 10% of the authors of papers appearing in the JPE were employed by the University of Chicago at the time of the publication. … Harvard and MIT [doctorates] alone account for about 50% of all papers published in the QJE."

[44] For example, a recent article (Zinovyeva and Bagues 2015) examines the centralized process for faculty promotion decisions in all academic disciplines at Spanish universities from 2002 – 2006. The evaluators were selected randomly from a pool of eligible evaluators. The article finds that candidates are approximately 50 percent more likely to be promoted when the committee includes, their thesis advisor, a coauthor, or a colleague.

[45] The slowing of editorial decisions by economics journals is studied by Ellison (2002).

[46] For example, the *B.E. Journal of Theoretical Economics* started as a four-tier family of journals (*Frontiers*, *Advances*, *Contributions*, and *Topics*), but subsequently eliminated the *Frontiers* tier and has now reduced to a two-tier system.

Internet search. Various journals have introduced a variety of mechanisms to incentivize referees to complete their reviews faster and to reduce publication lags.

Still, it is evident that the economics profession could do a lot to get its own house in order before taking on the outside world. And, while the stakes may seem smaller from innovating in economists' market institutions than in reforming a large urban school district or in improving the efficiency of a multi-billion dollar spectrum auction, the feasibility of implementing creative new designs is likely to be much greater.

# 6 Conclusion

In its relatively brief life, the field of market design has already become extremely influential. By developing practical allocation mechanisms and applying them to real-world problems, it has reshaped and improved markets involving energy products, telecommunications spectrum, school choice, organ transplantation, entry-level professional positions, and many other diverse areas of the economy, touching many lives in very positive ways.[47] Equally importantly for the progress of the field, practical policy concerns have broadened the research frontier with new sets of questions and methodological approaches. New challenges motivate bold and equally exciting future research directions.

Complementarities among goods and market participants emerged as a common challenge spanning most applications. Classic constructs such as the Vickrey-Clarke-Groves mechanism and Gale and Shapley's deferred acceptance algorithm satisfy most design objectives in substitutes environments; however, reasonably pessimistic theoretical conclusions and severe design challenges are introduced when complementarities are present. On the auction side, anomalies and vulnerabilities emerge in the VCG mechanism. On the matching side, a key issue that emerges in practice is that, when a mechanism fails to accommodate complementarities and package preferences, market participants may act outside of the mechanism to meet their demands. Tight theoretical limits for complementarities and naturally evolving solutions in the marketplace suggest that mechanisms could benefit from meeting the substitutes component of demands to the extent possible, leaving room supplementary to the mechanism or outside of the mechanism for market participants to accommodate their complementary demands. Such practical solutions may help participants to meet their demands for complex packages, thereby enhancing participation, as well as to help eliminate commitment problems in markets.

[47] Indeed, in a recent book (Siegfried 2010) consisting of case studies showing how economic research has improved economic and social conditions over the past half century by influencing public policy decisions, two of the twelve case studies were taken from market design.

Just as an extensive empirical auctions literature has developed in recent years, one can expect a parallel development in matching markets. There remains much to do in understanding demand in matching markets, and this will become increasingly possible as large data sets from areas including school choice become available. By bringing in theory and structural econometric modelling, a new data-driven approach may emerge. At the same time, research concerns are likely to expand beyond merely allocating a fixed set of products, to include product design as well, offering a broader perspective (and greater potential for improving) auction and matching markets.

Insights drawn from behavioral economics are likely to play increasingly important roles in market design. As in the past, behavioral considerations will continue to be incorporated in part by extensive use of experimental economics methods in tandem with market design. One can also expect that behavioral theories and psychological theories will take on growing importance in market design. In addition, there will be growing recognition of the significance of the computer interfaces that are used for implementing market mechanisms, and their design will more explicitly be directed toward exploiting or negating the relevant behavioral biases.

The substantive areas where market design is applied will undoubtedly expand with time. As we have discussed, the improvement of secondary markets and the design of the FCC incentive auction are new frontiers for theory and applications. Economists can also be expected to turn inward and to attempt to reform their own institutions. At the same time, the recent history of the field makes clear that some of the most exciting applications of market design in coming decades are likely to come in totally unexpected areas. Prior to the early papers on kidney exchange, who would have anticipated the large and constructive influence that market designers would have on an area so far afield from traditional economic inquiry as organ transplantation? Similarly, one can expect with some confidence that some of the most influential future applications of market design are not on anybody's radar screens today.

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