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Durable Goods Monopoly with Incomplete Information

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This article reconsiders the durable goods monopoly problem when the monopolist's marginal cost is private information. We show that the Coase Conjecture implies the *No Trade Theorem*: In any equilibrium in which the lowest-cost seller's initial offer approaches her marginal cost, the aggregate probability of trade must vanish. However, we also construct non-Coasean equilibria which approximate the unique outcome of the rental version of the same model. These (stationary) equilibria are comparatively efficient.

The results are equally applicable to the mathematically-equivalent problem of sequential bargaining with two-sided incomplete information where one party makes all the offers.

What are the consequences of monopoly in a market for a durable good? Present-day understanding of this question divides into two strands of thought. First, the monopolist may effectively lack any ability to credibly precommit on her future output. Under this scenario, the price will drop almost instantaneously to marginal cost, apparently yielding the efficient outcome normally associated with perfect competition.[1] Second, the monopolist may possess an explicit (e.g. leasing the good rather than selling[2]) or implicit (e.g. the maintenance of reputation[3]) device enabling her to adhere to an output restriction. Under such circumstances, price will be maintained at a non-competitively high level, suggesting an outcome fraught with all the inefficiency of classical monopoly.

The conclusions of the first paragraph rest on analysis of a standard model in which the monopolist's payoff function is taken to be common knowledge. A single firm with known constant marginal cost sells an infinitely-durable good to consumers with a known distribution of valuations for the good.[4] The firm has the ability to revise its posted price

1. This proposition was exposited by Ronald Coase (1972), who wrote that, with complete durability, "the competitive outcome may be achieved even if there is but a single supplier". The formal Coase Conjecture was developed in work by Bulow (1982), Stokey (1981), Fudenberg, Levine and Tirole (1985), and Gul, Sonnenschein and Wilson (1986).

2. Coase (1972) argued that leasing or other similar contractual arrangements might be required in order for a durable goods monopolist to attain a monopoly price. Judge Charles Wyzanski earlier recognized a similar point in *United States v. United Shoe Machinery Corp.*, 110 F. Supp. 295 (D. Mass. 1953), aff'd per curiam, 347 U.S. 521 (1954). In adopting the Government's proposed antitrust remedy of requiring that United Shoe offer for sale any machine that it leased, he wrote: "The merit of the Government's proposal is in its secondary impact. Insofar as United's machines are sold rather than leased, they will ultimately, in many cases, reach a second-hand market. From that market, United will face a type of substitute competition which will gradually weaken the prohibited market power which it now exercises".

3. In Ausubel and Deneckere (1989*a*), we constructed subgame-perfect equilibria that permit the durable goods monopolist to earn all levels of profits, including arbitrarily close to static monopoly profits, as the time interval between offers approaches zero. In these equilibria, the firm adheres to an output restriction in order to avoid developing a reputation as a monopolist who cuts prices.

4. The Coase Conjecture, interpreted as meaning that the outcome of durable goods monopoly converges to the competitive price and quantity, requires a distributional assumption which we have earlier termed "no gap": the demand curve is not completely inelastic at marginal cost. We will consciously restrict attention to the case of "no gap" in this article.

at frequent time intervals; consumers have rational expectations. Alternatively, one can imagine a monopolist possessing private information about cost. One might have hoped that the conclusions drawn from the standard analysis would be robust to the richer informational environment and that the usual common knowledge assumption represented inessential detail. The standard model would then be an elegant abstraction for studying the fundamental differences between monopolistic markets for durable and non-durable goods.

However, the thrust of the current article will be that the introduction of private cost information may substantially alter the two strands of thought in the opening paragraph. Suppose that the monopolist knows her marginal cost exactly, while consumers merely possess a probability distribution over possible marginal costs. It is important to observe that in this (or essentially any) specification with private information, there now become available two distinct avenues by which social surplus may be dissipated:

• *Conventional deadweight loss*: The monopolist may charge a price which exceeds her (true) marginal cost, and so some consumers who value the good at greater than the social cost will not consume the good.

• *Signaling cost*: The monopolist may delay her sales of the good, in order to credibly signal her cost, and so the consumption of the good over a period of time may be lost to society.

In the standard model, in which the monopolist's payoff was publicly known, the only potential source of waste was conventional deadweight loss. Eradicating conventional deadweight loss (through the usual Coasean logic) was tantamount to achieving social efficiency. By way of contrast, as soon as the monopolist's cost is made private information, signaling becomes a second potential source of waste. Moreover, the competitive outcome is now precluded by incentive compatibility considerations,[5] and there come to be tradeoffs between the two avenues for social loss. In particular, pricing above marginal cost is not so certain to lead to dismal inefficiency, if it has the offsetting benefit of enabling credible truth-telling without costly delay.

In the first part of this article, the usual relationship between the Coase Conjecture and efficiency is turned on its head. Suppose that, in the private cost model, the monopolist is unable to price above marginal cost. (We only require that, for the *lowest possible* realization of marginal cost, the monopolist is unable to credibly maintain a higher price.) We demonstrate a result which we call the No Trade Theorem: as the low-cost seller's initial price approaches marginal cost, the ex ante expected probability of trade converges to zero. The nature of our result is thus that, if gains from trade are not at all consumed by conventional deadweight loss, then they will instead be fully dissipated through signaling costs. An inability to charge higher than marginal cost, rather than implying the efficient outcome of the standard model, leads to disastrously inefficient consequences.

In the second part of this article, stationary sequential equilibria are constructed in the private cost model which grossly depart from the predictions of the Coase Conjecture. At the outset, it is useful to observe that the *rental* version of the durable goods monopoly model has a unique sequential equilibrium. In this equilibrium, the monopolist departs from marginal-cost pricing in the same way as a classic (non-durable goods) monopolist:

5. By the "competitive outcome", we mean the intersection of supply and demand curves *based on the true valuations of monopolist and consumers.* In the private cost model, the competitive outcome would then consist of the monopolist always offering *a price equal to her actual marginal cost* (for each possible realization of marginal cost). When the supports of the seller and buyer distributions are overlapping intervals, the reasoning of Myerson and Satterthwaite (1983) precludes the competitive outcome in the private cost model.

for every realization of the random draw, the firm charges the monopoly rental price (relative to her true marginal cost). While the *sales* market possesses a continuum of stationary sequential equilibria, the ones we focus on closely mimic the unique outcome of the rental market. Such *monopoly equilibria* utilize a natural reputational device: the consumers' incomplete information provides the monopolist with a credible means to hide her cost and hence not to reduce her price. Were the monopolist to unexpectedly cut her price, consumers would view this as a sign that her marginal cost was low and her resolve was weak, and they would accordingly expect even further price slashing in the future.

In the third and final part of this article, the usual link between conventional deadweight loss and inefficiency is modified. We demonstrate that the monopoly equilibria are relatively efficient. Indeed, under certain distributional assumptions, the appropriately-defined limit of the monopoly equilibria is optimal within a class of outcomes where the seller fully reveals her marginal cost to consumers before any trade occurs. The economic intuition for this optimality result can be stated quite neatly. In monopoly equilibria, *each seller type* finds herself able to charge her most preferred price. By itself, this creates substantial conventional deadweight loss. However, partially offsetting this loss, the high degree of pricing latitude leaves the seller with no reason whatsoever to distort her true marginal cost to consumers. Full revelation becomes credible, *without any costly delay* at all.

The conclusions of this article also bear on our understanding of bargaining, because of the close relationship between the durable goods monopoly model and the sequential bargaining game where one party makes all the offers. Gul and Sonnenschein (1988) have argued that there is a difficulty in matching the empirical predictions of sequential bargaining models with observed practice: "One hopes for a theory in which agents communicate their private information by revealing their willingness to delay agreement and that in a significant number of realizations agreement will occur only after some delay". However, the Coase Conjecture, when carried over to a context of bargaining under one-sided incomplete information, provides the counterfactual conclusion of "no delay"—the parties always instantaneously come to agreement, and "strikes" never occur.

By way of contrast, this article demonstrates that an opposite (but equally extreme and unpalatable) result comes about when the Coase Conjecture is adapted to bargaining under two-sided incomplete information. The parties delay, and delay, and further delay, without reaching agreement—strikes last essentially forever. However, this does not establish a general proposition that sequential bargaining models with two-sided incomplete information are doomed to predict disagreement. Our construction of the monopoly equilibrium demonstrates that there also exist sequential equilibria in which the Coase Conjecture fails, and so the bargaining may succeed.

We proceed to a description of the private cost model in the next section, followed by the three main parts of the article and a conclusion. Most proofs are relegated to an Appendix.

2. THE MODEL.

We consider a market for an infinitely-durable good which is demanded only in quantities of zero or one. The market is served by a single seller but contains a continuum of consumers. The monopolist's constant marginal (and average) cost of production, s, is private information; the monopolist knows s, but consumers treat it as a random variable with distribution function $F_1(\cdot)$. While each individual consumer's actual valuation for the good is common knowledge, consumers are atomistic[6] and different consumers have different valuations. The demand side of the market is thus characterized by a distribution function, $F_2(\cdot)$; for every b, $F_2(b)$ indicates the proportion of consumers whose valuations are less than or equal to b. As is usual in games of incomplete information, both distribution functions are common knowledge among all players. Throughout this article, we will assume that $F_1(\cdot)$ and $F_2(\cdot)$ are strictly monotone, absolutely continuous functions which share a common support.[7] There is then no further loss of generality in restricting attention to the case where supp $F_i=[0,1]$, for $i=1,2$. We will write $f_1(\cdot)$ and $f_2(\cdot)$ for the respective density functions of the two distributions.

The monopolist offers the durable good for sale at discrete moments in time, spaced equally apart. Let $z>0$ denote the time interval between successive offers; then sales occur at times $t=0, z, 2z, \dots, nz, \dots$. Within each period, the timing of moves is as follows: first, the monopolist names a price at which the good is made available to all consumers who have not previously purchased; then, all such consumers decide whether or not to buy. After a time interval z elapses, the play repeats. All participants are infinitely-lived and equally impatient, discounting time using the real interest rate $r>0$. Thus, if a consumer with reservation value b purchases the good at time t for the price p_t, he derives a net surplus of $e^{-rt}(b-p_t)$. Similarly, if m_t denotes the proportion of consumers purchasing at time t, then the monopolist's net present value of profits is given by $\sum_t e^{-rt}m_t(p_t-s)$, where the sum is taken over all times t such that $t=nz$ for some non-negative integer n. Consumers seek to maximize net surplus, and the monopolist seeks to maximize the net present value of profits.

While our article is cast almost entirely in the language of durable goods monopoly, it has implications for a closely-related model of bargaining. A single seller, with valuation s, makes repeated offers to a single buyer, with valuation b, for the sale of an indivisible object. The valuations are private information to the respective traders but are independently drawn from commonly-known distribution functions $F_1(\cdot)$ and $F_2(\cdot)$. While the models are not formally equivalent,[8] it should be observed that for every (actual) consumer in the durable goods monopoly model there is associated a (potential) buyer type in the bargaining formulation, and vice versa. Indeed, there is a one-to-one correspondence between equilibria of the durable goods monopoly game and the bargaining game.

At the start of each period n, the history H^n of the game consists of the sequence of the n previous prices offered and the set of consumers who accepted each of these

6. Each individual consumer has measure zero; moreover, we henceforth assume that players' strategies do not respond to deviations by a single consumer. The latter restriction reflects our quest for equilibria in which consumers act as price-takers. This restriction affects the equilibrium set even in the durable goods monopoly model where the seller's cost is known, as argued by Gul, Sonnenschein and Wilson (1986, p. 170).

7. The assumption of a common support has varying degrees of importance for the different results of this article. Adopt as notation that supp $F_1=[\underline{s},\bar{s}]$ and supp $F_2=[\underline{b},\bar{b}]$. Obviously, our results carry over to the case where $\underline{b}\leqq\underline{s}<\bar{b}\leqq\bar{s}$, as buyer types in the interval $[\underline{b},\underline{s})$ and seller types in the interval $(\bar{b},\bar{s}]$ can then be disregarded. This seems to be the only available generalization of our theorem on the optimality of monopoly in Section 5. To see this, consider examples of the form supp $F_2=[\underline{b},\bar{b}]$ and supp $F_1=[\underline{s},\underline{s}+\varepsilon]$, where $\underline{s}\in[\underline{b},\bar{b})$. Then, for ε sufficiently small, the constant price mechanism with $\pi=\underline{s}+\varepsilon$ clearly dominates the monopoly mechanism. Our construction of monopoly equilibria in Section 4 makes essential use of the weaker assumption that $\underline{b}\leqq\underline{s}<\bar{s}\leqq\bar{b}$. Indeed, our monopoly equilibria are sustained by optimistic beliefs coupled with reversion to Coasean pricing; as in the game where the seller's cost is publicly known (Ausubel and Deneckere (1989*a*)), such a reversion will deter the seller of type s only if $s\in[\underline{b},\bar{b}]$ (i.e. if there is no gap). Finally, our No Trade Theorem of Section 3 fully generalizes even to the case where $\underline{b}>\underline{s}$ and $\bar{s}\neq\bar{b}$ (see Corollary 1).

8. The models are not formally equivalent since, in the durable goods monopoly game, the seller literally *observes* the set of consumers who purchase. In contrast, in the bargaining game, the seller merely *infers* the set of types who should have purchased. The equilibrium paths of the two models are identical on account that we have explicitly assumed, in the durable goods formulation, that players' strategies do not respond to deviations by a single consumer—see footnote 6.

prices. A (pure) strategy for the monopolist then specifies which price to offer, as a function of the monopolist's own type and the previous history:

$$\sigma_s^n : [0, 1] \times H^n \rightarrow [0, 1].$$

Having observed history H^n and the seller's offer p^n in period n, consumers form a (common) posterior conjecture, $\mu_b^n(s \mid H^n, p^n)$, of the seller's marginal cost. Each consumer who has not previously purchased then decides whether or not to accept the current offer, according to his valuation, the pre-period history and the current price, using the strategy:

$$\sigma_b^n : [0, 1] \times H^n \times [0, 1] \rightarrow \{\text{accept, reject}\}.$$

We require that the consumers' strategies be measurable in their valuations (the first argument of σ_b^n). This ensures that the set of consumer acceptances in each period will be a measurable set, so that the monopolist's net present value of profits can be evaluated from any point in the game onward.

A *sequential equilibrium* is a pair of strategy profiles $\{\sigma_s^n, \sigma_b^n\}_{n=0}^{\infty}$ and a system of beliefs $\{\mu_b^n\}_{n=0}^{\infty}$ that satisfy sequential rationality and a weak notion of consistency. By consistency we will mean that the posterior beliefs conditioned on the history H^n must be computed using Bayes' rule whenever possible, and must remain within the support of the original distribution $F_1(\cdot)$.[9] Sequential rationality requires that, given any history and the induced posterior beliefs, strategies from then on must be optimal for all players. In order to ensure the existence of equilibrium, we will need to permit the monopolist to mix at any stage of the game. It should be clear how to extend the above definitions when behavioural strategies are allowed.

In analyzing the durable goods monopoly and the related bargaining problem, several authors (Gul, Sonnenschein and Wilson (1986), Gul and Sonnenschein (1988), Cho (1990)) have found it useful to further restrict attention to sequential equilibria in which players' strategies and updating rules are "stationary". Stationarity has been deemed attractive on simplicity grounds, since players base their actions on a short summary of the history. In the context of the present model, the *state* is described by: (a) the distribution of consumer valuations currently remaining in the market; and (b) the consumers' current beliefs about the monopolist's marginal cost. By a (strong) *stationary sequential equilibrium*, we will mean that agents' strategies and updating rules depend on the pre-period history of the game *only insofar as it is reflected in the current state.* As observed by Fudenberg, Levine and Tirole (1985), a sequential equilibrium satisfying this strong notion of stationarity often fails to exist. In a weak stationary sequential equilibrium, the above requirement is slightly relaxed to permit the monopolist's strategy to depend not only on the current state, but also on the price she charged in the previous period.

Consider now any Nash equilibrium of the above game which involves pure strategies along the equilibrium path,[10] and denote by $t(s, b)$ and $\pi(s, b)$ the equilibrium time and price at which the monopolist with marginal cost s trades with the consumer of valuation b. (If, in equilibrium, trade does not occur, define $t(s, b) = +\infty$ and let $\pi(s, b)$ be any value in $[0, 1]$.) Then honest reporting must be a Nash equilibrium in the direct revelation

9. The equilibria we will construct below also satisfy the additional consistency requirements of what Fudenberg and Tirole (1988) termed "perfect Bayesian equilibrium."

10. We do not wish to suggest that randomization cannot occur along the equilibrium path; in the event this occurs, the text here would need to be appropriately modified. However, it is useful to recall that existence results in this area only require us to resort to mixed strategies *off the equilibrium path* (see, for example, Gul, Sonnenschein and Wilson (1986) and Ausubel and Deneckere (1989*a*)).

game where players report their types to a mediator, who then directly implements trade at the times and prices specified by $t(\cdot,\cdot)$ and $\pi(\cdot,\cdot)$ evaluated at the reports s and b.[11] As in Ausubel and Deneckere (1989b), we may give the sequential mechanism $\{t, \pi\}$ a (static) direct mechanism interpretation by performing the transformation:

$$\begin{aligned} p(s, b) &= e^{-rt(s,b)} \\ x(s, b) &= e^{-rt(s,b)}\pi(s, b). \end{aligned} \tag{1}$$

Observe that $p(\cdot,\cdot)$ has the interpretation of a probability of trade and $x(\cdot,\cdot)$ has the interpretation of an expected payment so that $\{p, x\}$ is a direct bargaining mechanism as defined by Myerson and Satterthwaite (1983). Under the assumption that $p(\cdot,\cdot)$ and $x(\cdot,\cdot)$ are measurable and the convention that the integrals below are Lebesgue integrals, we may define the following functions:

$$\begin{aligned} &\bar{p}_1(s) = \int_0^1 p(s, v_2)f_2(v_2)dv_2, && \bar{p}_2(b) = \int_0^1 p(v_1, b)f_1(v_1)dv_1, \\ &\bar{x}_1(s) = \int_0^1 x(s, v_2)f_2(v_2)dv_2, && \bar{x}_2(b) = \int_0^1 x(v_1, b)f_1(v_1)dv_1, \\ &\bar{U}_1(s) = \bar{x}_1(s) - s\bar{p}_1(s), && \bar{U}_2(b) = b\bar{p}_2(b) - \bar{x}_2(b). \end{aligned} \tag{2}$$

Thus, $\bar{p}_1(s)$ may be viewed as the monopolist's (interim) probability of trade, $\bar{x}_1(s)$ may be viewed as her expected revenues, and $\bar{U}_1(s)$ may be viewed as her expected profits, given a marginal cost of s and a truthful report. The quantities $\bar{p}_2(b)$, $\bar{x}_2(b)$, and $\bar{U}_2(b)$ have corresponding interpretations for the consumer of valuation of b. Since $\{p, x\}$ is derived from a Nash equilibrium, it must be incentive compatible, i.e.

$$\begin{aligned} &\bar{U}_1(s) \geqq \bar{x}_1(s') - s\bar{p}_1(s'), && \forall s, s' \in [0, 1], \\ &\bar{U}_2(b) \geqq b\bar{p}_2(b') - \bar{x}_2(b'), && \forall b, b' \in [0, 1]. \end{aligned} \tag{3}$$

As usual, the direct mechanism $\{p, x\}$ must also be interim individually rational. However, since $\{p, x\}$ is derived from a Nash equilibrium of the durable goods monopoly game and since consumers' equilibrium acceptances are necessarily ex post individually rational in this game, we obtain the somewhat stronger individual rationality constraints:

$$\begin{aligned} \bar{U}_1(s) &\geqq 0, && \forall s \in [0, 1], \\ bp(s, b) - x(s, b) &\geqq 0, && \forall s, b \in [0, 1]. \end{aligned} \tag{4}$$

Henceforth, we will refer to any direct mechanism which satisfies inequalities (3) and (4) as an *incentive compatible bargaining mechanism* (ICBM).

One immediate benefit of introducing the notation of ICBM's is that we can invoke the results of Myerson and Satterthwaite (1983, Corollary 1) to conclude that no *Nash* equilibrium of the durable goods monopoly game under incomplete information can be

11. In the bargaining formulation of the model, both the seller and the buyer have "types", and we are invoking the Revelation Principle. In the durable goods monopoly formulation, it should be observed that consumers do not literally have types—rather, each consumer's valuation is common knowledge. It is nevertheless useful to think of each consumer as reporting his type to a mediator, since in every period the monopolist makes a single offer to consumers (e.g. the monopolist cannot perfectly price discriminate by making individualized offers to different consumers). The consumer incentive-compatibility condition in (3), below, can be seen to be equivalent to a requirement that each consumer optimize against the monopolist's price path.

ex post efficient. In the next section, the notation will also enable us to quickly establish that the Coase Conjecture implies a much more severe degree of inefficiency.

Before proceeding, it should be observed that the individual rationality and incentive compatibility constraints from the static mechanism clearly do not exhaust the implications of sequential equilibrium in the durable goods monopoly game. In particular, we will make use below of a somewhat tighter characterization:

Lemma 1. *Let* $\{p, x\}$ *be any ICBM induced by a sequential equilibrium of the durable goods monopoly game. Then* $p(s, \cdot)$ *is a (weakly) increasing function of* b*, for each* s*, and* $p(s, b) = 0 = x(s, b)$ *whenever* $b < s$*.*

3. THE COASE CONJECTURE AND THE NO TRADE THEOREM.

Any treatment of durable goods monopoly with private cost information must address the Coase Conjecture, which has been the main focus of prior analyses that assume the seller's cost to be common knowledge. This section studies the ramifications of the Coase Conjecture in the private cost model, finding that the tendency toward efficiency in the standard model is reversed.

The "Coase Conjecture" has generally been interpreted as the following proposition: for any $\varepsilon > 0$, there exists $\bar{z}(\varepsilon) > 0$ such that, whenever the time interval between offers is less than $\bar{z}(\varepsilon)$, the seller's *initial offer* is less than the seller's marginal cost plus ε. In the standard model, the set of stationary equilibria satisfies the Coase Conjecture.[12] Of course, for the Coase Conjecture to have any force in the private cost model, equilibria will need to be separating in the limited sense that the monopolist reveals her cost to the buyer before any trade occurs. Even then, and even if attention is restricted to stationary equilibria, reputational considerations not present in the standard model may dissuade a high-cost seller type from cutting her price à la Coase: any effort to reduce prices in the hope of accelerating sales could be interpreted by consumers as a sign that the seller's costs are lower than previously believed. Consumers might then refrain from purchasing, despite the lower price, rendering the price cut unprofitable.

However, in stationary separating equilibria, the *lowest-cost* monopolist would not be deterred by the prospect of ruining her reputation—it is already as bad as it can get. Consequently, in such equilibria, the low type's initial price converges toward marginal cost as the time interval between offers approaches zero. But as we prove in Theorem 1, below, even a Coase Conjecture *for the lowest-cost monopolist only* has disastrous implications: the outcome displays a virtually complete absence of trade.

12. It is important to note that our notion of stationarity differs slightly from the notion which has been used in previous Coase Conjecture studies. In the standard durable goods monopoly model (where marginal cost is common knowledge), the "state" would merely consist of the seller's current beliefs about remaining consumer valuations. Our definition of stationarity, specialized to this context, permits both the monopolist and consumers to condition on the state variable. In contrast, Gul, Sonnenschein and Wilson (1986) use a definition which in essence restricts attention to equilibria of the infinite-horizon game which are limits of equilibria of finite-horizon games. The monopolist is permitted to condition on the state variable; however, consumers, confined to "reservation price strategies," ignore even the state and decide whether to purchase solely on the basis of the current price.

The Gul-Sonnenschein-Wilson notion of stationarity implies the Coase Conjecture in the standard durable goods monopoly model. To our knowledge, it is an open question whether our notion of stationarity is by itself sufficient to imply the Coase Conjecture in the standard model. Gull and Sonnenschein (1988) provide additional (monotonicity) conditions which, taken together with the definition of stationarity in the current article, imply the Coase Conjecture.

Theorem 1 (The No Trade Theorem). *Consider any sequence $\{\sigma^n\}_{n=1}^{\infty}$ of sequential equilibria and $\{z^n\}_{n=1}^{\infty}$ of associated time intervals between offers, such that $z^n \downarrow 0$ and the initial offer of the lowest-cost seller type in σ^n converges to marginal cost. Let $\{p^n, x^n\}_{n=1}^{\infty}$ be the associated sequence of ICBM's. Then as $n \to \infty$, the ex ante expected probability of trade, $\int_0^1 \int_0^1 p^n(s, b) f_2(b) f_1(s) db ds$, converges to zero.*

Proof. Recall from Myerson and Satterthwaite (1983, equation (4)) that $\bar{U}_1^n(0) = \bar{U}_1^n(1) + \int_0^1 \bar{p}_1^n(s) ds$. From Lemma 1, $\bar{U}_1^n(1) = 0$ in any sequential equilibrium; by the Coase Conjecture hypothesis of the theorem, $\lim_{n \to \infty} \bar{U}_1^n(0) = 0$. Consequently, $\lim_{n \to \infty} \int_0^1 \bar{p}_1^n(s) ds = 0$. Now observe that:

$$\int_0^1 \int_0^1 p^n(s, b) f_2(b) f_1(s) db ds = \int_0^1 \bar{p}_1^n(s) f_1(s) ds. \tag{5}$$

Since $f_1(\cdot)$ is Lebesgue integrable, the right-hand side of equation (5) also converges to zero as $n \to \infty$, yielding the desired result. ||

It is easy to understand the economic forces which drive Theorem 1. If the lowest-cost monopolist earned approximately zero profits, but a higher-cost type had a substantial probability of trade, then the former could mimic the latter and thereby increase her profits. Incentive compatibility thus forces, in aggregate, a low probability of trade.

As far as "efficiency" and "delay" are concerned, the Coase Conjecture becomes a two-edged sword. It requires extreme efficiency (and no delay) for the lowest-cost monopolist. But, in order to prevent the lowest-cost monopolist from imitating higher-cost types, this necessitates extreme inefficiency (and prolonged delay) for essentially all types with marginal cost above the minimal level.

Theorem 1 obviously also applies to the bargaining interpretation of the model. One earlier (Cramton (1984)) and one contemporaneous (Cho (1990)) paper treat the extensive form where the seller makes all the offers and where there is a continuum of seller and buyer types. Both authors present equilibria which satisfy the hypothesis of Theorem 1 (under the distributional specification that the lowest seller valuation is contained in the support of the buyer distribution). In Cramton's equilibrium, the seller uses delay to credibly signal her strength. Low-valuation seller types make revealing offers early in the game, while high-valuation seller types pool with unrealistic offers (yielding zero sales) until revealing later in the game. After having revealed, the seller quickly skims through buyer valuations using a Coase Conjecture strategy (from the game where the seller's cost is commonly known). Thus, as the time between successive offers shrinks toward zero, the seller's initial revealing offer approaches her true valuation. Applying our Theorem 1, this means that every seller type (except the lowest) does not reveal until a time which gets pushed arbitrarily far into the future. In the limit, the expected delay becomes infinite.

By way of contrast, in Cho's equilibrium, the Coase Conjecture holds for the lowest-valuation seller only. Every seller type reveals her valuation in the initial period by making a separating offer which has a positive probability of acceptance. As the time between periods declines toward zero, the initial offer of the lowest seller type approaches her valuation. By Theorem 1, it must be the case that the initial (revealing) offers of all other seller types converge to the highest buyer valuation and that the subsequent price path becomes arbitrarily flat. In this way, essentially all trade is again deferred infinitely far into the future.

Since Theorem 1 has such dramatic consequences, and since Cramton's and Cho's articles do not require the traders' distribution functions to have a common support, it is instructive to inquire what happens if our distributional assumptions are relaxed.[13] Without loss of generality, we may keep supp $F_1=[0,1]$, and let the support of $F_2(\cdot)$ more generally be $[\underline{b},\bar{b}]$. Clearly, if $\underline{b}<0$, the conclusion of Theorem 1 still holds. If $\underline{b}>0$, and by analogy with the standard model where the seller's marginal cost is common knowledge, the Coase Conjecture would have the lowest seller's initial offer converging to $\underline{b}$ (Fudenberg, Levine and Tirole (1985), Gul, Sonnenschein and Wilson (1986)). Identical reasoning as above then requires all seller types above $\underline{b}$ to face arbitrarily long delay, implying gross inefficiency whenever $F_1(\underline{b})$ is relatively small. We summarize in:[14]

Corollary 1. *Suppose that supp* $F_1=[0,1]$ *and supp* $F_2=[\underline{b},\bar{b}]$. *Consider any sequence* $\{\sigma^n, z^n\}_{n=1}^{\infty}$ *such that* $z^n \downarrow 0$ *and the initial offer of the zero-cost seller type in* σ^n *converges to* $\max\{0,\underline{b}\}$. *Then the ex ante expected probability of trade satisfies* $\lim_{n\to\infty}\int_0^1\int_{\underline{b}}^{\bar{b}} p^n(s,b)f_1(s)f_2(b)\,db\,ds \leqq F_1(\max\{0,\underline{b}\})$.

When the supports of the seller and buyer distribution overlap (i.e. $\underline{b}<1$ in the above corollary), the results of Myerson and Satterthwaite (1983) already precluded the possibility of full ex post efficiency or, equivalently, no delay in trade. The value-added of our Corollary 1 is that whatever delay is necessitated for incentive compatibility reasons is compounded (to the extreme, when $\underline{b}\leqq 0$) when the lowest seller type is cursed by the Coase Conjecture. Of course, when the seller and buyer distributions do not overlap (i.e. when $\underline{b}\geqq 1$), Corollary 1 no longer provides any bound on efficiency. Indeed, in that case, full ex post efficiency is attainable by, roughly speaking, having all seller types pool at any price between 1 and $\underline{b}$.

4. MONOPOLY EQUILIBRIA.

Our motivation in this section is to discover an equilibrium of the durable goods monopoly model which is more appealing than an outcome of essentially no trade. The approach we follow is to temporarily turn to a model in which the monopolist leases, instead of sells, the durable good. We will see that this rental model has a unique equilibrium, which can then be replicated by sequential equilibria of the sales model.

For a brief interlude, consider a monopolistic *lessor* of a durable good. At the start of every period, the monopolist names a price at which she will lease the good until the next period. Each consumer then elects whether or not to rent. After a time interval of z elapses, the play repeats. The demand side of the market is now characterized by a function $H(b)=F_2[b/(1-e^{-rz})]$, which provides the distribution of consumers' one-period rental valuations. Observe that this distribution is equivalent to that of the sales model: if a consumer's valuation for renting the good one period equals $(1-e^{-rz})b$, then his net present value of owning the good in perpetuity equals $\sum_{n=0}^{\infty}e^{-nrz}(1-e^{-rz})b=b$.

13. If $F_1(\cdot)$ and $F_2(\cdot)$ have a common support, but are not absolutely continuous, an analogous result to Theorem 1 still holds. Let $G(\cdot)$ be the smallest concave function that dominates $F_1(\cdot)$. Then $G(\cdot)$ is an absolutely continuous distribution function satisfying $G(0)=F_1(0)$ (see Lemma 1 of Ausubel and Deneckere (1988), the working paper precursor to this article). Note that even if $F_2(\cdot)$ is not absolutely continuous, the calculations from Myerson and Satterthwaite (1983) apply. Recalling that $\bar{p}_1^n(\cdot)$ is a monotone function satisfying $0\leqq\bar{p}_1^n\leqq 1$, and denoting the density of $G(\cdot)$ by $g(\cdot)$, we then see that the rightmost expression of (5) can be bounded by $\bar{p}_1^n(0)G(0)+\int_0^1\bar{p}_1^n(s)g(s)ds$. Consequently, the ex ante expected probability of trade converges to $F_1(0)$, which equals zero provided that $F_1(\cdot)$ does not have a mass point at zero.

14. It should be observed that the Cramton (1984) and Cho (1990) equilibria do, in fact, have every seller type in $[0,\underline{b}]$ charging an initial price of $\underline{b}$ (in the limit as $n\to\infty$), so that the limiting bound of Corollary 1 is attained with equality.

It is easy to see that the model has a unique equilibrium.[15] In every period, the monopolist makes a single take-it-or-leave-it offer to consumers and therefore optimizes by offering the monopoly rental price, $\tilde{\pi}_R(s)$, corresponding to her privately-known marginal cost s. This is defined by:

$$\tilde{\pi}_R(s)=[1-e^{-rz}]\tilde{\pi}(s), \tag{6}$$

where $\tilde{\pi}(s)\in \arg\max_\pi\{(\pi-s)(1-F_2(\pi))\}$. Observe using eq. (6) that $\tilde{\pi}(s)$, the monopoly sales price, satisfies $\tilde{\pi}(s)=\sum_{n=0}^{\infty} e^{-nrz}\tilde{\pi}_R(s)$, so we see that the unique equilibrium of the rental game corresponds to an outcome path of the sales game where each seller type offers her monopoly sales price, $\tilde{\pi}(s)$, forever.

The idea behind the construction of our *monopoly equilibria* is to come as close as possible to replicating, in the sales model, the price paths implicit in the rental model. We will not be able to literally replicate the rental equilibrium: it has the property that the seller type with lowest marginal cost charges her monopoly price forever; whereas, in any sequential equilibrium of the sales model, the price charged by the lowest type must converge toward cost as the time t tends toward infinity. However, we can arbitrarily closely approximate the rental equilibrium in the sales model. For any given cost c close to zero, there exists a sufficiently large discount factor and a sequential equilibrium such that all seller types above c charge their monopoly prices forever.

Let c $(0<c<1)$ be an arbitrary choice of marginal cost and let λ $(\lambda>0)$ be an arbitrary choice of a rate of descent. A third value, $\hat{c}$, satisfying $0<\hat{c}<c$, will be endogenously determined below. We begin by specifying the monopolist's pricing behavior along the equilibrium path. The complete *menu of price paths*, and the types assigned to each path, are given by:

If $s\in[0,\hat{c}]$: Charge the price $\pi_0^\lambda(t)\equiv e^{-\lambda t}\tilde{\pi}(0)$, (7a)
for each $t=0, z, 2z, \ldots$
until such time that $\pi_0^\lambda(t)<s$;
Charge a price of 1 in all subsequent periods.

If $s\in(\hat{c}, c)$: Charge the price $\pi_c(t)\equiv\tilde{\pi}(c)$, (7b)
for each $t=0, z, 2z, \ldots$.

If $s\in[c, 1]$: Charge the price $\pi_s(t)\equiv\tilde{\pi}(s)$, (7c)
for each $t=0, z, 2z, \ldots$.

Verbally, equations (7) state that all seller types in the interval $[c, 1]$ charge the monopoly price corresponding to their true type forever. Seller types in $(\hat{c}, c)$ pool with type c in charging the monopoly price of type c forever. Meanwhile, seller types in the interval $[0, \hat{c}]$ pool by following the price path $\pi_0^\lambda(\cdot)$ so long as it remains profitable. However, for every type, there will come a time when sales on this descending price path become unprofitable (and continue to be unprofitable thereafter). At this time, the seller "drops off" the price path $\pi_0^\lambda(\cdot)$ and ceases to ever sell again.

Consumer behaviour, along the equilibrium path, is to optimize against the chosen price path, under the assumption that the monopolist has and will continue to follow (7). Thus, if consumers hear an initial offer of $\tilde{\pi}(s)$, where $s\in(c, 1]$, they infer that the seller's marginal cost equals s, and they purchase if and only if their valuation is at least

15. Uniqueness is nearly, but not quite perfectly, a description of the equilibrium set. For some distributions $F_2(\cdot)$ and some marginal costs s, there exist multiple solutions to $\arg\max_p\{(p-s)(1-F_2(p))\}$. However, for fixed $F_2(\cdot)$, the set of s leading to multiple monopoly prices has measure zero. Hence, the expected payoff of each player is unique. Henceforth, we will ignore this form of non-uniqueness.

$\tilde{\pi}(s)$. If consumers hear an initial offer of $\tilde{\pi}(c)$, they infer that the seller's marginal cost is distributed on the interval $(\hat{c}, c]$ (distributed according to a truncated version of $F_1(\cdot)$), and they purchase if and only if their valuation is at least $\tilde{\pi}(c)$. Thus, for each of these intervals of seller types, all purchases which ever occur happen in the initial period. Finally, if consumers hear an initial offer of $\tilde{\pi}(0)$, their optimization problem is made somewhat more complicated by the seller's dropping-off behaviour. The consumer with valuation b optimizes by selecting the time t (a non-negative integer multiple of z) which maximizes $e^{-rt}G_1[\pi_0^\lambda(t)]\{b-\pi_0^\lambda(t)\}$, where $G_1(s)\equiv F_1(s)/F_1(\hat{c})$, for $s\leqq \hat{c}$, and $G_1(s)\equiv 1$, for $s>\hat{c}$ (i.e. $G_1(\cdot)$ is $F_1(\cdot)$ truncated to the sub-interval $[0, \hat{c}]$). Indeed, an optimizing consumer confronted with a current offer of $\pi_0^\lambda(t)$ must take into account that the subsequent price $\pi_0^\lambda(t+z)$ will only be offered with probability $G_1[\pi_0^\lambda(t+z)]/G_1[\pi_0^\lambda(t)]$, and hence must use an implicit discount factor of $e^{-rz}G_1[\pi_0^\lambda(t+z)]/G_1[\pi_0^\lambda(t)]$ between times t and $t+z$.

We complete the description of the sequential equilibrium by specifying the beliefs and actions which follow deviations. First, suppose that the monopolist *detectably* deviates from the pricing strategy of equations (7). Consumers immediately update their beliefs to $s=0$ (with probability one) and maintain those beliefs forever after. Furthermore, consumers adopt the consumer strategy from a stationary sequential equilibrium of the game where the monopolist's cost is commonly known to equal 0. Thus, following a detectable deviation, consumers believe they are facing the lowest-cost seller, and expect her to follow a Coase Conjecture price path. Meanwhile, the monopolist maximizes against this consumer strategy. Second, suppose that the monopolist *undetectably* deviates from her equilibrium strategy. Obviously, consumers cannot have yet updated their beliefs. The monopolist calculates her payoff from continuing to impersonate the type(s) she has thus far mimicked, compares that to her payoff from optimally detectably deviating (and drawing the revised consumer beliefs), and follows whichever course is preferable.

We wish to show that, for every fixed c, the above strategies and updating rules form a sequential equilibrium for sufficiently small λ and z. It will suffice for us to establish three facts for sufficiently small λ and z:

Fact 1. *The monopolist correctly self-selects from the menu of price paths in the initial period, provided that she expects to adhere to her chosen price path in all subsequent periods (and that initial detectable deviations are excluded).*

Fact 2. *A monopolist type in the interval $[\frac{1}{2}\hat{c}, 1]$ obtains zero continuation profits following any detectable deviation and, thus: (a) does not detectably deviate in the initial period; and (b) cannot do better than adhering to her initial choice of price path in all subsequent periods.*

Fact 3. *A monopolist type in the interval $[0, \frac{1}{2}\hat{c})$ obtains sufficiently small continuation profits following any detectable deviation that: (a) an initial choice of $\pi_0^\lambda(\cdot)$ dominates any other initial selection; and (b) given an initial choice of $\pi_0^\lambda(\cdot)$, continued adherence to $\pi_0^\lambda(\cdot)$ is optimal.*

These three facts are proved in the Appendix.

The monopoly equilibria which we have just constructed can also easily be seen to be *stationary* equilibria. Each price path $\pi_s(\cdot)$, $s\in[c, 1]$, is a constant price path, surely lending stationarity. Along $\pi_0^\lambda(\cdot)$, there are sales in every period in which price is at least $\hat{c}$; and buyer beliefs about the seller evolve in every period that price is less than $\hat{c}$

(as sellers "drop off" every period). If the seller has undetectably deviated, and if her strategy instructs her to continue to undetectably deviate, the same argument as above shows stationarity. If the seller ever detectably deviates, this fact is thereafter reflected in the state, as only histories with detectable deviations lead to buyer beliefs that $s=0$. Strategies after a detectable deviation are certainly stationary, as buyers merely use reservation price strategies and sellers optimize against them.

Before moving on to the next section, the reader may desire some explanation as to why it was possible to introduce reputation effects into the equilibrium without also adding non-stationarities. The key insight is that reputation in this model operated directly through beliefs: the high-cost seller refrained from price-cutting, lest consumers come to believe she is actually a low-cost seller. Since beliefs about the seller's cost are themselves a component of the "state", the maintenance of reputation did not require players' strategies to utilize the history beyond its reflection in the current state.

5. THE EFFICIENCY OF MONOPOLY.

The Cramton (1984) and Cho (1990) equilibria of the sales game share a property which we may call *revelation before trade*: all information that the monopolist ever reveals is revealed before she makes her first trade. Revelation-before-trade equilibria have a simple and fairly natural structure, in which both the seller and buyer signal their bargaining strengths through delay in the agreement. Thus, seller types who are less patient (e.g. those who expect larger gains from trade) reveal their costs and begin to trade sooner than seller types who are more patient. Moreover, once revelation has occurred, buyer types also separate over time according to their degree of impatience. Thus, after seller revelation, an outcome ensues of the (well-understood) game in which the seller's cost is publicly known.[16]

Revelation-before-trade equilibria, while attractive on intuitive grounds, also admit a wide variety of outcomes. To see this, it is convenient to re-phrase the revelation-before-trade property in terms of direct mechanisms:

Definition 1. A *seller-first mechanism* is an ICBM $\{p, x\}$ satisfying:

(i) $bp(s, b)-x(s, b) \geqq bp(s, b')-x(s, b'), \quad \forall s, b, b' \in [0, 1]$; and
(ii) $p(s, b)=0 \quad \text{whenever } b<s, \quad \forall s, b \in [0, 1]$.

Condition (i) requires that the buyer report truthfully, given prior knowledge of the seller's type.[17] Thus, seller-first mechanisms are ICBM's which retain their incentive compatibility even if consumers are fully informed of the monopolist's type.[18] Condition (ii) is necessarily satisfied if the mechanism is derived from a sequential equilibrium of

16. An additional justification for focusing on the set of ICBM's with the revelation-before-trade property is that the full set of ICBM's *cannot* be replicated by sequential equilibria of the usual extensive form games (see Ausubel and Deneckere (1988)). In particular, even many ICBM's which satisfy the requirements of Lemma 1 are not induced by sequential equilibria of the durable goods monopoly game. Nor is there available any neat characterization of the ICBM's which are induced by sequential equilibria (unlike the game where the seller's cost is publicly known; see Ausubel and Deneckere (1989*b*)). However, if we require the revelation-before-trade property, then we restrict attention to a set of ICBM's whose interesting elements *can* be replicated by sequential equilibria with an intuitive structure (see the paragraph following Example 2, below).

17. One way to interpret these static mechanisms is to imagine that the mediator, rather than soliciting simultaneous reports, first asks the seller to report her type within earshot of the buyer. Only then does the mediator require the buyer to reveal his valuation.

18. The reader should not be misled by the observation that some equilibria with the revelation-before-trade character are "pooling" in the usual sense (see Example 1 below). Any of our pooling equilibria would be undisturbed if, in an initial period, seller types were to fully separate by making (essentially) revealing non-serious offers, and then continue as before.

the seller-offer game (see Lemma 1). The following theorem provides a complete characterization of the set of seller-first mechanisms (we use the notation $p_s(r) \equiv p(s, r)$ and $x_s(r) \equiv x(s, r)$):

Theorem 2. *In any seller-first mechanism, $p(s, \cdot)$ is monotone non-decreasing in b (for fixed s) and $x(s, b) = \int_0^b r\,dp_s(r)$. Conversely, if $\{p, x\}$ is an ICBM such that $p(s, \cdot)$ is monotone non-decreasing in b, $x(s, b) = \int_0^b r\,dp_s(r)$ and $p(s, b) = 0 = x(s, b)$ whenever $b < s$, then it is a seller-first mechanism.*

Observe that seller-first mechanisms satisfy the requirements of Lemma 1 and, hence, are natural candidates for outcomes of the seller-offer game. Observe also that the Riemann-Stieltjes integral for $x(s, \cdot)$ and the monotonicity of $p(s, \cdot)$ imply that $\pi(s, b) = x(s, b)/p(s, b)$ is monotone non-decreasing in b, so that each seller type indeed charges a declining sequence of prices. Finally, note that the set of all seller-first mechanisms has a simple structure: it is a non-empty, closed, convex subset of the set of all ICBM's.

The richness of the set of seller-first mechanisms can be illustrated with the following two examples:

Example 1. Constant-price mechanisms.

$$p^1(s, b) = \begin{cases} 1 \\ 0 \end{cases} \quad x^1(s, b) = \begin{cases} \pi & \text{if } 0 \leqq s \leqq \pi \text{ and } \pi \leqq b \leqq 1 \\ 0 & \text{otherwise.} \end{cases}$$

This type of mechanism has been studied by Chatterjee and Samuelson (1983) in the context of a static, simultaneous-bid bargaining game.

Example 2. *The monopoly mechanism.*

$$p^2(s, b) = \begin{cases} 1 \\ 0 \end{cases} \quad x^2(s, b) = \begin{cases} \tilde{\pi}(s) & \text{if } \tilde{\pi}(s) \leqq b \leqq 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\tilde{\pi}(s) \in \arg\max_\pi \{(\pi - s)(1 - F_2(\pi))\}$. This mechanism has been studied by Chatterjee and Samuelson (1983) in the context of a take-it-or-leave-it, single-offer bargaining game (with $F_2(b) = b$). We have already encountered the mechanism as the unique equilibrium of the rental game; the "monopoly equilibria" of the sales game come arbitrarily close to replicating it.

In Ausubel and Deneckere (1988), the working paper precursor to this article, we proved that *any* seller-first mechanism can be replicated by sequential equilibria of the durable goods monopoly and bargaining games, provided that it satisfies an additional sequentiality condition (which we termed "sequentially seller-first")[19] and provided that certain technical conditions hold. The general construction is analogous to, but more intricate than, the development of the monopoly equilibria in Section 4.

19. The reason why an additional sequentiality condition (above and beyond Definition 1) is necessary for a seller-first mechanism to be replicated is that the monopolist cannot be held to a strategy which calls for all future sales to be at below cost. The continuation profits from such a strategy are negative, whereas the worst available punishment is "no subsequent trade." Thus, the seller of type s cannot be deterred from mimicking type $s' < s$ until such time that her price is supposed to drop below s, and then refusing to trade thereafter.

Let $\hat{b}(s', s) \equiv \inf\{b \in [0, 1]: p(s', b) > 0$ and $x(s', b)/p(s', b) \geqq 0\}$, and define $\tilde{U}_1(s', s) = \int_{\hat{b}(s',s)}^1 [x(s', b) - sp(s', b)] f_2(b)\,db$.

A *sequentially seller-first* mechanism is a seller-first mechanism satisfying $\tilde{U}_1(s, s) \geqq \tilde{U}_1(s', s)$, $\forall s, s' \in [0, 1]$. Observe that Examples 1 and 2 are sequentially seller-first. For a seller-first mechanism which is *not* sequentially seller-first, see Ausubel and Deneckere (1988, p. 25).

The least efficient (sequentially) seller-first mechanism is the zero-price mechanism (i.e. the constant-price mechanism with $\pi = 0$);[20] we have already seen (in Theorem 1) that it arises from sequential equilibria satisfying the Coase Conjecture. Surprisingly, under some mild distributional conditions, the monopoly mechanism of Example 2 is the most efficient mechanism in this class:

Theorem 3. *Suppose $F_1(\cdot)$ is convex and C^2, $F_2(\cdot)$ is concave and C^2, and $v_2 - [1 - F_2(v_2)]/f_2(v_2)$ is an increasing function.*[21] *Then the most efficient seller-first mechanism is the monopoly mechanism.*

Theorem 3 should be contrasted with the case where the seller's cost is commonly known, in which all equilibria of the seller-offer game are necessarily seller-first, and for which the mechanism that maximizes the gains from trade corresponds to marginal cost pricing. The reason that monopoly pricing fares reasonably well in the case of private cost information is that, although the allocative losses due to the presence of monopoly pricing are large, the efficiency losses due to the truth-telling constraints are minimal. By way of contrast, the Coase Conjecture equilibria minimize allocative losses due to pricing above marginal cost, but maximize efficiency losses from the truth-telling constraint.

6. CONCLUSION.

The existing literature on durable goods monopoly suggests that the monopolist has compelling incentives to seek alternative arrangements to the outright sale of durable goods, in order to avert the Coase Conjecture. Unfortunately, such alternative arrangements may result in social loss, on two separate grounds: first, they probably increase the cost of providing the underlying stream of services; and second, they enable the seller to engage in monopoly pricing. The conclusion which one might be tempted to draw is that, as a matter of antitrust policy, society should bar firms from establishing these alternative arrangements.

For example, Coase (1972) and Bulow (1982, 1986) emphasized that a durable goods monopolist might choose to rent rather than sell, even though this creates a potential moral hazard problem—renters are likely to devote less effort to maintaining the durable good than are buyers. Equally, the monopolist seller has an incentive to reduce the durability of the good, in order to put her closer to the position of a renter. Bulow also argued that the monopolist seller might inefficiently underinvest in capital, selecting a technology with low fixed cost and high marginal cost in order to increase the price she could credibly charge. Policy makers have, at least at times, taken this type of argument seriously; for example, in the case of *United States v. United Shoe Machinery Corp.*, the government advocated and the court ordered that the defendant firm offer for sale any machine which it leased.

One straightforward implication of the current article is that, when private cost information is introduced into a durable goods monopoly, the firm may no longer need to concoct alternative arrangements to pure sales. There exist appealing equilibria of the sales model which come close to replicating the monopolist's most favoured outcome. A second implication is that it is no longer so clear that society will wish to ban such

20. It shares this dubious distinction with the constant-price mechanism with $\pi = 1$.

21. The monotonicity of $v_2 - [1 - F_2(v_2)]/f_2(v_2)$ guarantees that the profit function is quasi-concave for each seller type, so that the monopoly mechanism is uniquely defined. (See also footnote 15.) This assumption is standard in the literature on ex ante efficiency (see Myerson and Satterthwaite (1983) and Williams (1987)).

alternative arrangements, in the event that the monopolist needs to resort to them. Whereas the Coase Conjecture leads to the essentially complete dissipation of gains from trade in the private cost model, the rental equilibrium provides a surprisingly good outcome.[22]

But this is not to suggest that monopoly is the best of all possible worlds. Under durable goods monopoly with private cost, ex post efficiency is unattainable: it is necessary to allow the monopolist to price above marginal cost in order to dissuade her from engaging in costly signaling behaviour. By way of contrast, the presence of multiple firms makes full efficiency possible, as rivals may compete both in the price they charge and in the cost information they reveal.[23] Even in a market for durable goods, a policy promoting genuine competition may be the best prescription for enhancing welfare.

APPENDIX

Proof of Lemma 1. The monotonicity of $p(s, \cdot)$ in b follows directly from the successive-skimming lemma (the fact that high-valuation consumers purchase no later than low-valuation consumers) in Cramton (1984, p. 582). Suppose that $p(s, b) > 0$ for some $b < s$. By equation (4), the price $\pi(s, b)$ at which s sells to b satisfies:

$$\pi(s, b) = x(s, b)/p(s, b) \leqq b < s, \tag{A.1}$$

meaning that s loses on every sale at the price $\pi(s, b)$. Using the successive-skimming lemma, there also are no future profitable sales to be made, so s could profitably deviate by refusing to sell (i.e., making an unrealistically high offer) at $t(s, b)$ and all later times. Since $x(s, b) = p(s, b)\pi(s, b)$, this also establishes that $x(s, b) = 0$ whenever $b < s$. ||

Proof of Facts 1, 2 *and* 3. Fact 1 can be established for arbitrary choice of c, given sufficiently small λ and z. Define $W(\hat{c}, \lambda, z; s)$ to be the expected payoff to seller s from following the price path $\pi_0^\lambda(\cdot)$ (until $\pi_0^\lambda(t) < s$), when consumers believe that $\hat{c}$ is the cutoff used by seller types in selection rule (7). Also define $W_c(s)$ to be the expected payoff to seller s from optimizing among the remaining price paths $\{\pi_r(\cdot)\}_{r\in[c,1]}$. Seller type s will select between $\pi_0^\lambda(\cdot)$ and $\{\pi_r(\cdot)\}_{r\in[c,1]}$ according as $W(\hat{c}, \lambda, z; s) > W_c(s)$ or $W(\hat{c}, \lambda, z; s) < W_c(s)$. Observe that, for sufficiently small λ and z, $W(\hat{c}, \lambda, z; 0) > W_c(0)$, whereas $W(\hat{c}, \lambda, z; c) < W_c(c)$. Furthermore, since $W(\hat{c}, \lambda, z; \cdot)$ is continuous and convex while $W_c(\cdot)$ is linear (on the domain $[0, c]$), there exists a unique seller type $\gamma(\hat{c})$ so that all $s \in [0, \gamma(\hat{c}))$ strictly prefer $\pi_0^\lambda(\cdot)$ and all $s \in (\gamma(\hat{c}), c]$ strictly prefer $\{\pi_r(\cdot)\}_{r\in[c,1]}$. Correct self-selection requires that $\gamma(\hat{c}) = \hat{c}$; since the map $\gamma(\cdot)$ is easily shown to be continuous, the Brouwer fixed point theorem guarantees the existence of such a $\hat{c}$. This establishes Fact 1.

The existence of a stationary sequential equilibrium in the game where the monopolist's cost is commonly known to equal zero (from which the consumers' strategy after a detectable deviation is specified) is guaranteed, for every $F_2(\cdot)$ and every $z > 0$, by Theorem 4.2 of Ausubel and Deneckere (1989*a*). In any such equilibrium, we may define the "Coase price" (denoted $\pi_{\text{Coase}}(b)$) to be the price which the seller is supposed to charge, and the "choke price" (denoted $\pi_{\text{choke}}(b)$) to be the supremum of all prices which induce positive sales, when the state (i.e. the highest remaining consumer valuation) is b. To complete the argument, let us assume that there exist positive constants L, M and α such that $Mb^\alpha \leqq F_2(b) \leqq Lb^\alpha$ for all $b \in [0, 1]$. This condition assures that, in the game where the monopolist's cost is commonly known, the uniform version of the Coase Conjecture (Theorem 5.4 of Ausubel and Deneckere (1989*a*)) holds for all stationary equilibria. This theorem implies that, for every $\varepsilon > 0$, there exists $\bar{z}(\varepsilon) > 0$ such that $\pi_{\text{Coase}}(b) < \varepsilon b$ and $\pi_{\text{choke}}(b) < \varepsilon b$, for all $b \in (0, 1]$, whenever

22. Of course, one does not need to resort to private cost information in order to reach the first conclusion. In earlier work (Ausubel and Deneckere (1989*a*)), we demonstrated that even with complete information there exist appealing equilibria of the sales model which come close to replicating the monopolist's most favoured outcome. The current article, however, should help to persuade adherents of *stationary* equilibria that the durable goods monopolist may avert the Coase Conjecture.

The second conclusion makes more essential use of the presence of private cost information. Indeed, in the standard model, society certainly *does* wish to interfere with arrangements intended to circumvent the precommitment problem.

23. We do *not* intend to suggest that full ex post efficiency becomes *inevitable* when actual or potential competitors are introduced. Collusive equilibria may also exist—see Ausubel and Deneckere (1987) for similar reasoning when cost is commonly known.

$0<z<\bar{z}(\varepsilon)$.[24] Hence, there exists $\bar{z}'>0$ such that $\pi_{\text{choke}}(b)<\frac{1}{2}\hat{c}b$ and $\pi_{\text{choke}}(b)<W(\hat{c},\lambda,z;\frac{1}{2}\hat{c})-W_c(\frac{1}{2}\hat{c})$, for all $b\in(0,1]$, whenever $0<z<\bar{z}'$.

Observe that the sales price following a detectable deviation in state b is forever after bounded above by $\pi_{\text{choke}}(b)$. Thus, the previous paragraph trivially establishes Facts 2a and b, since $\pi_{\text{choke}}(b)<\frac{1}{2}\hat{c}$ for all $b\in[0,1]$, and so any sales by a seller type in $[\frac{1}{2}\hat{c},1]$ after a detectable deviation occur at below cost. It also leads to Fact 3a, as follows. The best alternative to the initial-period equilibrium action, for a seller type $s\in[0,\frac{1}{2}\hat{c})$, is to follow path $\pi_c(\cdot)$ for one period and then detectably deviate (as $\pi_c(\cdot)$ generates no sales after the initial period). But profits from this deviation are bounded by $W_c(s)+\pi_{\text{choke}}(b)$, which is less than $W(\hat{c},\lambda,z;s)$ since $W_c(\frac{1}{2}\hat{c})+\pi_{\text{choke}}(b)<W(\hat{c},\lambda,z;\frac{1}{2}\hat{c})$. Thus, the seller would have done better by initially choosing $\pi_0^\lambda(\cdot)$.

The only deviations which remain to be excluded are of the form: some $s\in[0,\frac{1}{2}\hat{c})$ initially follows $\pi_0^\lambda(\cdot)$ but deviates in some subsequent period. Define $V(b;s)$ to be the continuation profits to the monopolist from following $\pi_0^\lambda(\cdot)$ when the current state is b and the seller's type is s. We will now demonstrate that there exist K_1, $K_2>0$ such that:

$$V(b;s)\geqq K_2b^\alpha(K_1b-s). \tag{A.2}$$

Begin by observing that, if all seller types $s\in[0,\hat{c}]$ initially follow price path $\pi_0^\lambda(\cdot)$ and if type s drops off in the first period that price falls below s, then every buyer $b\in[0,1]$ selects an optimal time to purchase by solving:

$$\max_t\{F_1[\min\{\hat{c},\tilde{\pi}(0)e^{-\lambda t}\}]e^{-rt}(b-\tilde{\pi}(0)e^{-\lambda t})\},\quad \text{where } t=0,z,2z,\ldots. \tag{A.3}$$

For each b, define $\underline{t}(b)$ by $\tilde{\pi}(0)e^{-\lambda\underline{t}(b)}=b$ and $\bar{t}(b)=\arg\max_t\{e^{-rt}(b-\tilde{\pi}(0)e^{-\lambda t})\}$. Observe, in (A.3), that b purchases no earlier than time $\max\{0,\underline{t}(b)\}$ and no later than time $\max\{0,\bar{t}(b)+z\}$. Suppose now that b_0 is the current state. Then the current time is at least $\max\{0,\underline{t}(b_0)\}$. Define $b_1=(M/2L)^{1/\alpha}b_0$, and observe that the time at which b_1 purchases is no later than $\max\{0,\bar{t}(b_1)+z\}$. Consequently, the seller discounts sales to the interval $[b_1,b_0)$ using a factor no less than $e^{-r[\bar{t}(b_1)-\underline{t}(b_0)+z]}=w\delta(M/2L)^{r/\alpha\lambda}$, where $w\equiv[r/(r+\lambda)]^{r/\lambda}$. The probability that b is contained in $[b_1,b_0)$ is bounded below by $F(b_0)-F(b_1)\geqq Mb_0^\alpha-Lb_1^\alpha=(M/2)b_0^\alpha$. Meanwhile, all sales to the interval $[b_1,b_0)$ occur at prices no less than $\tilde{\pi}(0)e^{-\lambda\max\{0,\bar{t}(b_1)+z\}}=\min\{\tilde{\pi}(0),[r/(r+\lambda)]b_1e^{-\lambda z}\}$. Note there exists $K_1>0$ such that $K_1b_0<\min\{\tilde{\pi}(0),\ [r/(r+\lambda)]\times(M/2L)^{1/\alpha}b_0e^{-\lambda z}\}$ whenever $0<z<\bar{z}'$. Hence, a lower bound on the continuation profit to type s when the state is b_0 is given by: $V(b_0;s)=we^{-r\bar{z}'}(M/2L)^{r/\alpha\lambda}(M/2)b_0^\alpha[K_1b_0-s]$. Simplifying notation by writing $K_2\equiv we^{-r\bar{z}'}(M/2L)^{r/\alpha\lambda}(M/2)$ and writing b in place of b_0 completes the derivation of inequality (A.2).

Now let $\bar{z}''(0<\bar{z}''<\bar{z}')$ be sufficiently small that $\pi_{\text{choke}}(b)<\frac{1}{2}K_1b$ and $\pi_{\text{Coase}}(b)<(\frac{1}{2}K_1K_2/L)b$ for all $b\in(0,1]$, whenever $0<z<\bar{z}''$. First, consider any pair (b,s) such that $s\geqq\pi_{\text{choke}}(b)$. When the highest remaining consumer valuation is b, seller s earns non-negative continuation profits by remaining on price path $\pi_0^\lambda(\cdot)$ (with the usual stopping rule). Deviation triggers Coase expectations, nets zero profits to seller s, and is hence deterred. Second, suppose $s<\pi_{\text{choke}}(b)$. Since $s<\frac{1}{2}K_1b$ and by (A.2), $V(b;s)>(\frac{1}{2}K_1K_2)b^{1+\alpha}$. Profits from deviation are bounded above by $\pi_{\text{choke}}(b)F_2(b)<(\frac{1}{2}K_1K_2)b^{1+\alpha}$, so deviation is again deterred, demonstrating that Fact 3*b* holds. ||

Proof of Theorem 2. Observe that Definition 1(i) implies that:

$$bp(s,b)-x(s,b)\geqq bp(s,b')-x(s,b'),$$

and (A.4)

$$b'p(s,b')-x(s,b')\geqq b'p(s,b)-x(s,b),$$

$\forall s,b,b'\in[0,1]$. Adding these inequalities yields $(b'-b)p(s,b')\geqq(b'-b)p(s,b)$. Thus, $p(s,b')\geqq p(s,b)$ if $b'\geqq b$, establishing the monotonicity of $p(s,\cdot)$. Also from (A.4), we see that:

$$b'[p(s,b')-p(s,b)]\geqq x(s,b')-x(s,b)\geqq b[p(s,b')-p(s,b)]. \tag{A.5}$$

Since $p(s,r)$ is monotone, the Riemann-Stieltjes integral $\int_0^b r\,dp_s(r)$ is well defined, and (A.5) implies that $x(s,b)=x(s,0)+\int_0^b r\,dp_s(r)$. Definition 1(ii) and inequality (4) together then imply that $x(s,0)=0$, yielding the desired formula for $x(s,\cdot)$.

24. Theorem 5.4 of Ausubel and Deneckere (1989*a*) directly states this fact for $\pi_{\text{Coase}}(\cdot)$. The observation that $b-\pi_{\text{choke}}(b)=e^{-rz}[b-\pi_{\text{Coase}}(b)]$ establishes the same fact for $\pi_{\text{choke}}(\cdot)$.

Also observe that $x(s, b')-x(s, b)=\int_b^{b'} r dp_s(r) \geqq b[p(s, b')-p(s, b)]$, by the monotonicity of $p(s, \cdot)$, immediately implying the converse. ||

Derivation of the result of Theorem 3.

Maximizing efficiency over the set of seller-first mechanisms is a potentially daunting task. Lemma 2 greatly simplifies the search. Before stating the lemma, let us define $p(\cdot, \cdot)$ to be a *0-1 mechanism* if there exists a non-decreasing boundary $\phi(\cdot)$ such that $p(s, b)=0$ for $b<\phi(s)$ and $p(s, b)=1$ for $b \geqq (s)$. We have:

Lemma 2. *Suppose* $0 \leqq p(s, b) \leqq 1$ *maximizes*:

$$f(p) \equiv \int_0^1 \int_0^1 (v_2 - v_1) p(v_1, v_2) f_2(v_2) f_1(v_1) dv_2 dv_1, \tag{A.6}$$

subject to the constraint:

$$h(p, s) \equiv \int_0^1 t_2 p(s, t_2) f_2(t_2) dt_2 - \int_0^1 \int_0^{v_2} p(s, t_2) dt_2 f_2(v_2) dv_2$$
$$- \int_s^1 \bar{p}_1(t_1) dt_1 - s\bar{p}_1(s) = 0, \quad \text{for all } s \in [0, 1]. \tag{A.7}$$

Suppose also that $p(\cdot, \cdot)$ *satisfies the requirements of a* $0-1$ *mechanism. Then, the mechanism* $\{p, x\}$, *with* $x(s, b)=\int_0^1 r dp_s(r)$, *maximizes the gains from trade among all seller-first mechanisms.*

Proof. In any ICBM, $\bar{x}_1(s)=\bar{U}_1(s)+s\bar{p}_1(s)=\int_s^1 \bar{p}_1(t_1)dt_1+s\bar{p}_1(s)$. Furthermore, by Theorem 2, in a seller-first mechanism $x(s, b)=\int_0^b r dp_s(r)=bp(s, b)-\int_0^b p(s, r)dr$. Hence we have $\bar{x}_1(s)=\int_0^1 t_2 p(s, t_2) f_2(t_2) dt_2 - \int_0^1 \int_0^{v_2} p(s, t_2) dt_2 f_2(v_2) dv_2$. Thus, constraint (A.7) must hold for any seller-first mechanism. Since equation (A.6) gives an expression for gains from trade, maximizing (A.6) subject to (A.7) is the desired optimization problem. Finally, the observation that $\{p, x\}$ satisfies the requirement of a seller-first mechanism completes the proof. ||

Proof of Theorem 3. We will show that there exists a function $\rho(s)$ of bounded variation on [0, 1] such that the 0–1 mechanism with boundary $\tilde{\pi}(s)$ satisfying:

$$\tilde{\pi}(s) - s - \frac{1-F_2(\tilde{\pi}(s))}{f_2(\tilde{\pi}(s))} = 0 \tag{A.8}$$

maximizes the Lagrangian $\mathscr{L}(p, \rho)=f(p)+\int_0^1 h(p, t_1)d\rho(t_1)$ subject to $0 \leqq p(\cdot, \cdot) \leqq 1$. The sufficiency theorem of Luenberger (1969, p. 220) then implies that $p(\cdot, \cdot)$ solves the infinite-dimensional linear program (A.6)-(A.7). Observe first that we may rewrite $\mathscr{L}$ as:

$$\mathscr{L}(p, \rho) = \int_0^1 \left(\int_0^1 \left[v_2 - v_1 - \frac{\rho(v_1)-\rho(0)}{f_1(v_1)} \right] p(v_1, v_2) f_1(v_1) dv_1 \right.$$
$$\left. + \int_0^1 \left[v_2 - v_1 - \frac{1-F_2(v_2)}{f_2(v_2)} \right] p(v_1, v_2) d\rho(v_1) \right) f_2(v_2) dv_2. \tag{A.9}$$

Now let $\rho(0)=0$ and, for $v_1>0$, let $\rho(v_1) \equiv f_1(v_1)[\tilde{\pi}(v_1)-v_1]$. Then (except at $v_1=0$, where $\rho(\cdot)$ has a mass point) the coefficient of $p(v_1, v_2) f_1(v_1) f_2(v_2) dv_1 dv_2$ in $\mathscr{L}(p, \rho)$ becomes:

$$v_2 - \tilde{\pi}(v_1) + \left(v_2 - v_1 - \frac{[1-F_2(v_2)]}{f_2(v_2)} \right) \left(\frac{f_1'(v_1)}{f_1(v_1)} [\tilde{\pi}(v_1) - v_1] + \tilde{\pi}'(v_1) - 1 \right). \tag{A.10}$$

Notice that, evaluated at the point $(v_1, \tilde{\pi}(v_1))$, the expression in (A.10) is equal to zero. Next, we will argue that its derivative with respect to v_2 is nonnegative. This will imply that the mechanism described above maximizes $\mathscr{L}$ and, hence, establish its optimality. Now, the derivative of (A.10) is given by:

$$1 + [2 + \omega(v_2)] \left(\frac{f_1'(v_1)}{f_1(v_1)} [\tilde{\pi}(v_1) - v_1] + \tilde{\pi}'(v_1) - 1 \right), \tag{A.11}$$

where $\omega(v_2)$ is implicitly derived from $(d/dv_2)\{v_2 - [1-F_2(v_2)]/f_2(v_2)\} \equiv 1 + \omega(v_2) > 0$. Concavity of F_2 implies that $\omega(v_2) \leqq 0$. Also, convexity of $F_1(\cdot)$ and $\tilde{\pi}(v_1) \geqq v_1$ implies that (A.11) $\geqq 1 + [2 + \omega(v_2)][\tilde{\pi}'(v_1) - 1]$. Since

$\tilde{\pi}'(v_1) = 1/[2+\omega(\tilde{\pi}(v_1))]$, and since $-2<\omega(v_2)\leqq 0$, we may bound the latter expression again to obtain (A.11) $> 1+[2+\omega(v_2)][-\frac{1}{2}] = -\frac{1}{2}\omega(v_2) \geqq 0$. This establishes the desired result. ||

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