Economics 603 Professor Lawrence Ausubel
Fall 2018–2019

Problem Set 7

1.

(a) Solve for the Nash equilibrium of the Cournot model of oligopoly ***for n firms*** with constant marginal (and average) costs all equaling c. Justify your answer.

(b) Suppose that the game is modified so that each firm has to pay a fixed cost of F in order to enter the market (and produce any positive quantity). Show that nobody's behavior is modified from part (a), provided that F is less than each firm's equilibrium profit.

(c) If the fixed cost F exceeds each firm's equilibrium profit (with n firms in the market), observe that at least one firm would have been better off if it had not entered the market. Assuming that there are no barriers to entry other than the fixed cost, and that there is an unlimited number of potential firms, endogenously determine the number of firms that will end up producing (and the market price), for any given $F > 0$.

(d) What happens in the limit as $F \to 0$?

2.

(a) Consider a Cournot game with two firms. The market inverse demand function is given by $P(Q) = 120 - Q$, where $Q = q_1 + q_2$ and q_i is the quantity produced by firm i. Each firm's total cost is quadratic, given by $C_1(q_1) = q_1^2$ and $C_2(q_2) = q_2^2$, respectively.

Determine the Nash equilibrium, and compute the associated payoffs.

(b) Now suppose that the two firms in a duopoly ***are permitted to explicitly collude*** by forming binding agreements with one another. The market inverse demand function is given by $P(Q) = 120 - Q$, where $Q = q_1 + q_2$ and q_i is the quantity produced by firm i. Each firm's total cost is quadratic, given by $C_1(q_1) = q_1^2$ and $C_2(q_2) = q_2^2$, respectively. Finally, assume that the firms have equal bargaining power.

Determine the quantities which the firms would agree upon, and compute the associated payoffs.

3. Solve for the Nash equilibrium of the Bertrand model of oligopoly ***for* 3 *firms*** with constant marginal (and average) costs ***equaling c_1, c_2 and c_3, respectively, where $c_1 > c_2 > c_3$.*** Justify your answer.

4. Consider the following static game. There are four players (i = 1,2,3,4). Each player i simultaneously and independently selects a strategy $s_i \in [0,10]$. Each player gets a benefit related to all of the players' choices of s_j, but incurs a cost related to her own s_i. In particular, the payoff to each player i is given by:

$$U_i(s_1, s_2, s_3, s_4) = s_1 + s_2 + s_3 + s_4 - \tfrac{1}{2} s_i^2 .$$

(a) Is the game solvable by elimination of strictly dominated strategies? If so, solve it by elimination of strictly dominated strategies. If not, indicate why it is not solvable by elimination of strictly dominated strategies. In either case, also determine *all* of the Nash equilibria of the game, and fully justify your answer.

(b) Calculate the ***socially-optimal*** outcome of the game, and explain why the social optimum is ***not*** an equilibrium.

5. The sealed-bid, double auction under complete information.

Suppose the following rules and payoffs of a game. There are two players: a seller and a buyer. There is one good, which is currently owned by the seller. The seller values the good at zero; the buyer values the good at one. Both players have linear utilities, so if they were to trade the good at price p, the seller would realize a payoff of p and the buyer would realize a payoff of $1 - p$. If there is no trade, each player realizes a payoff of zero.

Simultaneously and independently, the buyer writes down a bidding price, b, chosen from the closed interval [0,1], and the seller writes down an asking price, a, chosen from the closed interval [0,1]. They each hand the slips of paper to a referee. If the bid exceeds the ask (i.e., if $b \geq a$), then trade occurs at the average price (i.e., $p = ½ (a + b)$). If the bid is less than the ask (i.e., if $b < a$), then trade does not occur.

This is a game with a large number of pure-strategy, Nash equilibria. Find ***all of them***.

6. Demonstrate that problem 1.3 (p. 49) of Gibbons is almost exactly equivalent to the previous problem (#5) in this problem set. Solve Gibbons' problem 1.3.

7. In each of the following two-player games, what strategies survive iterated elimination of strictly-dominated strategies? What are the pure-strategy Nash equilibria?

(a)

Player II

Player I		L	C	R
	T	0 , 2	4 , 3	3 , 1
	M	1 , 2	2 , 0	2 , 1
	B	2 , 4	3 , 6	0 , 3

(b)

Player II

Player I		L	C	R
	T	1 , 3	5 , 4	4 , 2
	M	2 , 2	3 , 2	3 , 1
	B	3 , 5	4 , 3	1 , 4

8. Consider a Bertrand game with two firms. The market demand is given by $Q = 1 - P$, where $P = \min\{p_1, p_2\}$ and p_i is the price charged by firm i. Each firm's total cost is quadratic: $C_i(q_i) = q_i^2$, where q_i is the quantity sold by firm i.

   (a) One obvious structure for a Nash equilibrium of this game is a symmetric equilibrium where $p_1 = p_2$ and the two firms divide the market equally. Determine *all* such Nash equilibria.

   (b) Another potential structure for a Nash equilibrium of this game is that one firm charges a lower price than the other firm and the firm with the lower price makes all the sales. Determine all Nash equilibria with this structure, if such an equilibrium exists; or provide a clear and complete proof why no such equilibrium exists.