Final Exam

Answer all four questions, each in a *different* exam book.
Write your student number on each exam book and include your name on the book for Question 1.
Also write your name at the top of this question sheet, and please return this question sheet at the same time that you turn in the exam books.
This is a "closed everything" exam. Write legibly, show your work, and justify your responses.

1. [25 points] **The Game of Chicken**

Consider the two-player static game of complete information known as "Chicken". Two drivers of cars are heading directly towards each other. They each have two strategies: Drive Straight (D) or Swerve (S). If both players Swerve, they are safe but embarrassed. If one player Swerves and one player Drives Straight, the player who Drives Straight wins the game and the player who Swerves loses face. However, if both players Drive Straight, they have a head-on collision. The payoff matrix is:

	D	*S*
D	0, 0	7, 2
S	2, 7	6, 6

(a) [6 points] Find all Nash equilibria (pure-strategy and mixed) of this static game. Calculate the expected payoff for each player in the mixed-strategy Nash equilibrium.

(b) [5 points] Now consider a correlated equilibrium mediated by a traffic signal. The signal tells each player privately whether to "D" or "S". The probability distribution over recommendations is:

- (S, S) with probability 1/3;
- (D, S) with probability 1/3; and
- (S, D) with probability 1/3.

Verify that this constitutes a correlated equilibrium. That is, check that if Player 1 is told to "S", he has no incentive to switch to "D", and if told to "D", he has no incentive to switch to "S" (under the assumption that Player 2 follows the signal's recommendation).

(c) [10 points] Find the correlated equilibrium that maximizes the sum of the players' expected payoffs.

(d) [4 points] Does this correlated equilibrium of part (c) yield a higher total surplus than every public randomization over the Nash equilibria? Explain your answer carefully.

2. [25 points] **A Tragedy of Group Work**

Two students are working together on a project which, if successful, will yield each of them a utility of 1. The probability of success equals the product of their individual efforts, $e_i \in [0,1]$ ($i = 1,2$). Specifically, each player seeks to maximize her payoff $u_i(e_1, e_2) = e_1 e_2 - c e_i^2$, where $c > 0$ is a constant representing the disutility of effort.

(a) [7 points] Solve for all pure-strategy Nash equilibria, for all $c > 0$.

(b) [3 points] Calculate the levels of effort that would maximize the sum of their payoffs. For what values of c are the Nash equilibrium efforts below the socially optimal levels?

(c) Now suppose that student 1's section of the project can be written by ChatGPT, raising the floor of her work and making her "effective" effort $\frac{1+e_1}{2}$. The probability that the project is successful is now $\left(\frac{1+e_1}{2}\right)e_2$.

(i) [6 points] Find all interior pure-strategy Nash equilibria, where both students exert efforts $0 < e_i < 1$, for i = 1,2.

(ii) [5 points] Find the remaining pure-strategy Nash equilibria, where a student exerts either no effort or full effort.

(iii) [4 points] Compare the set of equilibria to what you found in part (a) and explain why it changes in this way.

3. [25 points] **An All-Pay Auction with a Rebate**

Consider a sealed-bid auction for a single item, in which two bidders (i = 1, 2) participate. Each bidder is risk neutral, with utility that is quasilinear in money. Bidder i's valuation, denoted by v_i, is private information and the v_i (i = 1, 2) are drawn from i.i.d. random variables that are each uniformly distributed on the interval [0,1]. After observing her own valuation v_i, each bidder i (i = 1, 2) simultaneously and independently submits a sealed bid $b_i \in [0,1]$. The bidder i who submitted the highest bid b_i wins the item. (All ties are resolved by a fair randomizing device.)

The payment rule is a variation on a standard all-pay auction. Both bidders—the winner and the loser—pay the amounts of their bids. However, the loser receives a rebate equaling two-thirds (2/3) of her bid. Hence, the loser's net payment is only one-third (1/3) of her bid.

(a) [15 points] Solve for a symmetric, Bayesian-Nash equilibrium of this game. Explain and fully justify your answer.

(b) [4 points] Provide a complete statement of the Revenue Equivalence Theorem,

(c) [6 points] Does the Revenue Equivalence Theorem apply to the comparison between this "all-pay auction with a rebate" and

(i) the standard all-pay auction?

(ii) the sealed-bid first-price auction?

Justify and explain your answers to both (i) and (ii).

4. [25 points] **A Repeated Cournot Game with Stochastic Demand**

In part (a), we will consider a static Cournot duopoly game ***G*** with stochastic demand, in which each of the two firms has an average and marginal cost of 3. First, a random variable is drawn that determines whether demand is high (H) or low (L). It is commonly known that the probability of H is θ and that the probability of L is $1 - \theta$, where $0 < \theta < 1$. Second, the two firms learn whether demand is H or L, and after learning this fact, they simultaneously and independently select their quantities, which are denoted by q_1 and q_2, respectively.

When demand is high, the inverse demand is given by $P_H(Q) = 9 - Q$, where $Q = \sum_{i=1}^{2} q_i$. When demand is low, the inverse demand is given by $P_L(Q) = 6 - Q$.

(a) [5 points] Find the Nash equilibrium of this game, as well as the expected profit of each firm evaluated before the level of demand (H or L) is realized.

In the rest of this problem, we will consider the infinitely repeated version of ***G***, in which each firm applies a common discount factor $\delta \in (0,1)$ between periods.

(b) [6 points] Graph the "Folk Theorem region" associated with:

i. The game ***G*** evaluated after nature has drawn high stochastic demand (H);

ii. The game ***G*** evaluated after nature has drawn low stochastic demand (L); and

iii. The game ***G*** evaluated in expectation before nature has drawn stochastic demand, for the special case of $\theta = \frac{1}{2}$.

(c) [4 points] Fully specify a pair of trigger strategies for a main equilibrium path in which the firms equally split joint monopoly profits in this repeated game.

(d) [7 points] Find the critical discount factor $\hat{\delta}$ such that the trigger strategies of part (c) form a subgame-perfect equilibrium, for general θ. Explain and justify your work.

(e) [3 points] How does $\hat{\delta}$ depend on θ? Explain intuitively why θ has this effect on firms' choices.