Economics 603 Professor Lawrence Ausubel
Fall 2023–2024

Final Exam

Answer all four questions, each in a *different* exam book.
Write your student number on each exam book and include your name on the book for Question 1.
Also write your name at the top of this question sheet, and please return this question sheet at the same time that you turn in the exam books.
Write legibly, show your work, and justify your responses.

1. [25 points] **A Three-Player Static Game**

Consider the following three-player static game. Each player i (i = 1, 2, 3) simultaneously and independently selects a strategy s_i from the closed interval [0, 10]. The payoff to each player i is given by:

$$U_i(s_1, s_2, s_3) = (s_1 - 5)(s_2 - 5)(s_3 - 5), \text{ for } i = 1, 2, 3.$$

(a) Find all of the pure-strategy Nash equilibria of this game. [<u>*Hint*</u>*: The game has a lot of Nash equilibria.*] Provide a clear justification for your response.

(b) Find at least two mixed-strategy Nash equilibria of this game in which every player i $(i = 1, 2, 3)$ places a strictly positive probability on exactly two strategies, $s_i \in [0,10]$ and $s_i' \in [0,10]$ (where $s_i' \neq s_i$). In addition, explain rigorously why each of the combinations of mixed strategies that you have provided in your answer is a Nash equilibrium.

2. [25 points] **An Infinitely-Repeated Game**

Consider the following two-player static game of complete information, denoted $\mathbf{G_1}$:

		Player II: L	C	R
Player I	T	7 , 2	2 , 1	0 , 3
	M	0 , 2	3 , 6	2 , 4
	B	0 , 1	4 , 3	0 , 2

(a) Find all of the Nash equilibria of the static game $\mathbf{G_1}$. Justify your response.

Parts (b) – (d) concern the infinitely-repeated version of $\mathbf{G_1}$ (with discount factor $\delta \in (0,1)$).

(b) Sketch (graphically) and write (algebraically) the "Folk Theorem region" for the infinitely-repeated version of static game $\mathbf{G_1}$.

(c) Write down a pair of trigger strategies that would provide average payoffs to the two players of approximately (5, 4), for discount factors near one. Specify the trigger strategies precisely.

(d) Determine the critical discount factor, $\hat{\delta}$, such that the trigger strategies of part (c) form a subgame perfect equilibrium (SPE), i.e., find the lowest $\hat{\delta}$ such that these strategies are an SPE for all discount factors δ such that $\hat{\delta} \leq \delta < 1$.

3. [25 points] **Correlated Equilibrium**

Consider the following two-player static game of complete information, denoted $\mathbf{G_2}$:

	L	R
T	5, 5	3, 6
B	6, 3	0, 0

(a) Find all of the Nash equilibria of the above game $\mathbf{G_2}$.

(b) In part (b), we shall consider correlated equilibria, limiting attention to correlated equilibria that are based on public randomizing devices. (An example of a public randomizing device is whether the closing Dow Jones Industrial Average is an even number or an odd number.) Determine the set of expected payoffs associated with all of the correlated equilibria of $\mathbf{G_2}$ that are based on public randomizing devices. Explain and fully justify your answer.

(c) In part (c), we shall allow all correlated equilibria, including those relying upon mediated communication. Write the optimization problem determining the symmetric correlated equilibrium of $\mathbf{G_2}$ that maximizes the sum of the players' expected payoffs—and solve for the optimum. Provide some intuition why this is the optimum and verify that it does strictly better than the correlated equilibrium described in part (b).

4. [25 points] **Stackelberg Duopoly with One-Sided Incomplete Information**

Consider duopoly games in which the (inverse) aggregate demand function is given by $P(Q) = 24 - Q$. Player 1 has a constant marginal and average cost of:

$$c_1 = \begin{cases} 2, & \text{with probability } \frac{3}{4}, \\ 6, & \text{with probability } \frac{1}{4}. \end{cases}$$

Player 1 always observes the realization of c_1 before she moves, but Player 2 does not observe the realization of c_1 (and he knows only its objective probability distribution).

Player 2 has a constant marginal and average cost of $c_2 = 4$ (which is common knowledge).

If Player 1 produces quantity q_1 and Player 2 produces quantity q_2, their payoffs are given by:

$$\pi_1(q_1, q_2) = q_1\left[P(q_1 + q_2) - c_1\right] \text{ and } \pi_2(q_1, q_2) = q_2\left[P(q_1 + q_2) - c_2\right].$$

(a) Consider the Stackelberg game with the following sequencing of moves: First, Player 1 selects her quantity, q_1, and Player 2 gets to see Player 1's choice of q_1. Second, Player 2 selects her quantity, q_2. Find the perfect Bayesian equilibrium (PBE) of this game.

(b) Consider the Stackelberg game with the following sequencing of moves: First, Player 2 selects her quantity, q_2, and Player 1 gets to see Player 2's choice of q_2. Second, Player 1 selects her quantity, q_1. Find the perfect Bayesian equilibrium (PBE) of this game.

(c) Explain the differences between the perfect Bayesian equilibria of parts (a) and (b).