Final Exam

Answer all four questions, each in a *different* exam book.
Write your student number on each exam book and include your name on the book for Question 1.
Also write your name at the top of this question sheet, and please return this question sheet at the same time that you turn in the exam books.
Write legibly, show your work, and justify your responses.

1. [25 points] **The Game of Whether or Not to Wear a Mask**

Consider a game with the following story behind it: A long time ago in a galaxy far, far away, there was a pandemic on the planet Tatooine. Each of the N inhabitants of Tatooine was faced with the choice of whether or not to wear a mask. While the masks by themselves were only slightly inconvenient, for unknown reasons, any inhabitant of Tatooine received strong disutility from wearing a mask if they learned that at least two other inhabitants of the planet went without a mask. Meanwhile, masks were only partially effective in protecting the wearer from catching the virus, but they were very effective in preventing the wearer from spreading the virus.

To be more precise, consider the following static game of complete information. The strategy set is {Wear a Mask, Do Not Wear a Mask}. The expected payoff to player i equals:

20, if player i does not wear a mask and all other players wear masks;

19, if player i wears a mask and all other players wear masks;

11, if player i does not wear a mask and exactly one other player does not wear a mask;

10, if player i wears a mask and exactly one other player does not wear a mask;

$-K^2 - 400$, if player i wears a mask and K other players ($K \geq 2$) do not wear masks; and

$-2K^2$, if player i does not wear a mask and K other players ($K \geq 2$) do not wear masks.

(a) Let $N \geq 25$. Is this game solvable by iterated elimination of dominated strategies? Provide a clear and concise justification for your response.

(b) Let $N \geq 25$. Does this game have a symmetric Nash equilibrium in pure strategies? If so, find it, and if not, provide a clear and rigorous argument that none exist.

(c) Let $N \geq 25$. Does this game have an asymmetric Nash equilibrium in pure strategies? If so, find ***all*** of them, and if not, provide a clear and rigorous argument that none exist.

(d) Let $N \geq 25$. Does this game have a Nash equilibrium in strictly mixed strategies? If so, provide a clear and rigorous argument that one exists—but you are ***not*** being asked to calculate one. If not, provide a clear and rigorous argument that none exist.

2. [20 points] **An Asymmetric First-Price Auction Game with Discrete Bids**

Consider the following sealed-bid auction game for a single item. There are two risk-neutral bidders, named Bidder 1 and Bidder 2, whose valuations are drawn from independent random variables. Bidder 1's valuation is uniformly distributed on the interval [0,1]. **However, Bidder 2's valuation is uniformly distributed on the interval [0, 2].** After observing its own valuation for the item, each of the two bidders simultaneously and independently submits a bid from the two-element set $\{0, \frac{2}{3}\}$. That is, each bidder is only allowed to submit a bid of zero or two-thirds. The high bidder wins the item and pays the amount of its bid (i.e., it is a first-price auction). Ties are resolved by the toss of a fair coin.

Solve for the Bayesian-Nash equilibrium of this first-price auction, fully explaining your work.

3. [25 points] **Cournot Duopoly with Bargaining**

Let ***G*** denote the standard static Cournot duopoly game. The inverse demand curve is given by $P = \max\{0, 12 - Q_1 - Q_2\}$, where Q_i ($i = 1, 2$) denotes firm i's quantity. Each firm's marginal and average cost is assumed to equal 0.

(a) [5 points] Suppose that firm 1's profit in ***G*** equals π_1. What strategy pair (Q_1, Q_2) of ***G*** generates the maximum profit for firm 2 (while generating π_1 for firm 1)? (By answering this question for all relevant π_1, you will have rigorously found the Pareto frontier for ***G***.)

(b) [10 points] Consider the infinitely-repeated version of ***G***, with discount factor $\delta \in (0, 1)$.

(i) What is the Folk Theorem region? Please graph the region clearly and accurately.

(ii) What discount factor(s) would support the average payoff pair of (18, 18) as a trigger strategy equilibrium?

(c) [10 points] Finally, consider an alternating-offer bargaining game in which firm 1 and firm 2 successively propose strategy pairs (Q_1, Q_2) for game ***G***. What are the subgame perfect equilibrium strategies? What are the subgame perfect equilibrium payoffs?

To be more precise: In the first period, firm 1 proposes strategy pair (Q_1, Q_2), and firm 2 decides whether to accept this proposal. If firm 2 accepts this proposal, the two firms are required to produce accordingly in game ***G***, the payoffs of game ***G*** are realized, and the game ends. If firm 2 rejects this proposal, the game proceeds to the second period, in which firm 2 proposes strategy pair (Q_1, Q_2), and firm 1 decides whether to accept this proposal. If firm 2 accepts this proposal, the two firms are required to produce accordingly in game ***G***, the payoffs of game ***G*** are realized but discounted by δ, and the game ends. And so on. The common discount factor is $\delta \in (0, 1)$.

4. [30 points] **A Hotelling Game with Endogenous Location**

Let two firms, $i = A, B$, be located at $L_A \in [0,1]$ and $L_B \in [0,1]$, respectively on the Hotelling line. Consumers are uniformly distributed on the Hotelling line and indexed by their location $x \in [0,1]$. Each consumer demands exactly one unit of the product. Consumer x's utility from buying product $i \in \{A, B\}$ at price p_i is $V_i = v - p_i - t|x - L_i|$, where v is a consumer's reservation value for the product, t measures the degree of horizontal differentiation, and $|\cdot|$ denotes absolute value. The marginal and average costs of each firm are normalized to 0.

Suppose both firms set their prices simultaneously. Consumers buy from the firm whose product generates the highest utility. Ties are resolved randomly. The quantity sold by a firm generally equals the length of the interval of consumers who buy from that firm. However, in the event that the two firms locate at the same point and charge the same price, they divide the market equally.

(a) [8 points] Suppose that $L_A = 0$ and $L_B = 1$.

(i) For a given pair of prices (p_A, p_B), which consumers buy from firm *A*?

(ii) What are the Nash equilibrium prices (limiting attention to pure strategies)?

(b) [10 points] Suppose that $L_A = 0$ and $L_B = ½$. Redo the analysis of part (a).

(c) [12 points] Consider the location choice of the two firms before price competition occurs. Assume that a firm can only choose to locate at any one of the three points 0, ½, and 1 on the Hotelling line. The game proceeds in two periods. In the first period, firm *A* and firm *B* each choose their own locations simultaneously and independently. In the second period, after the firms' location choices are publicly observed, the two firms set their prices simultaneously and independently. What equilibrium locations are chosen? [*Hint*: In addition to finding all pure-strategy equilibria, construct a mixed-strategy equilibrium in which each firm selects locations 0 and 1 with equal probabilities and also selects location ½ with positive probability.]