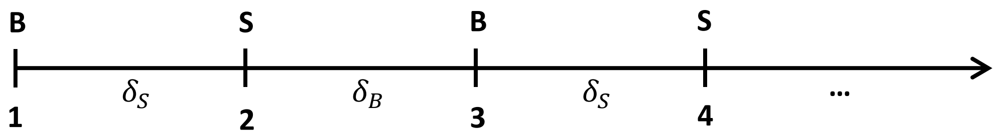
Economics 603 Professor Lawrence Ausubel
Fall 2021–2022

# Final Exam

**Answer all four questions, each in a *different* exam book.**
**Write your student number on each exam book and include your name on the book for Question 1.**
**Write legibly, show your work, and justify your responses.**

1. [20 points] **Alternating-offer bargaining with different offer preparation times**

Consider the following dynamic bargaining game between one seller and one buyer of an item. The item is commonly known to be worth 0 to the seller and worth 1 to the buyer. The buyer proposes a price at the beginning of the game. The seller can either accept the price, or reject and start preparing a counteroffer. The delay created by the seller's preparation time translates into a discount factor of $\delta_S \in (0,1)$ **for both the seller and buyer**. Similarly, after the seller proposes a price, the buyer can either accept the price, or reject and start preparing a counteroffer. The buyer's preparation time translates into a discount factor of $\delta_B \in (0,1)$ **for both the seller and buyer**. **Assume that the buyer's initial offer occurs at the very start of the game, with no associated discounting.**




To be clear, if the buyer proposes a price of $p_1$ at the beginning of the game and the seller accepts, the buyer's payoff is $1 - p_1$ and the seller's payoff is $p_1$. If the seller rejects and makes a counteroffer, the discount factor $\delta_S$ applies. If the buyer accepts the price proposed by the seller, denoted by $p_2$, the buyer's payoff is $\delta_S(1 - p_2)$ and the seller's payoff is $\delta_S p_2$. If an agreement is reached in the third round at a price of $p_3$, the buyer's payoff is $\delta_S\delta_B(1 - p_3)$ and the seller's payoff is $\delta_S\delta_B p_3$. And so on.

(a) [12 points] Find a subgame perfect equilibrium (SPE) of the bargaining game. Fully specify the buyer's and the seller's equilibrium strategies.

(b) [8 points] How does the seller's preparation time affect the buyer's equilibrium payoff? That is, would the buyer benefit more from bargaining with a slow seller or a fast seller? Provide some intuition for your answer.

2. [30 points] **The vaccination game: Omicron edition**

Suppose that there are *n* players, each of whom are assumed to be expected utility maximizers, and who simultaneously and independently decide whether or not to receive booster shots of a vaccine. The cost to a player of receiving the booster shot is $c > 0$, while the cost of getting sick with COVID is $s > c$. Assume that players who receive the booster shot will not get COVID. The probability of getting COVID, conditional on not receiving the booster shot, equals $\phi$, where $\phi$ is the proportion of a player's opponents who are unvaccinated. More precisely, if $v$ is the number of other players who get vaccinated, then $\phi = \left[1 - \frac{v}{n-1}\right]$, and an unvaccinated player's expected disutility equals $\phi s$.

(a) Characterize all of the pure strategy Nash equilibria of this game.

(b) For the special case of $c = 10$, $s = 100$, and $n = 101$, what are the possible numbers of players who are vaccinated in a pure strategy Nash equilibrium? Find all of the pure strategy Nash equilibria of this game and determine the number of pure strategy Nash equilibria.

(c) What happens to the proportion of vaccinated players as $s$ approaches $\infty$ (holding other parameters fixed)? What happens to the proportion of vaccinated players as $c$ approaches 0?

3. [25 points] **An infinitely-repeated game**

Let **G** denote the following static game of complete information. There are two players ($i$ = 1,2). Each player $i$ simultaneously and independently selects a strategy $s_i \in [0,4]$. The payoffs to the two players are given by:

$$U_1(s_1, s_2) = (s_2)^2 - (s_1 - 2)^2 \text{ and } U_2(s_1, s_2) = (s_1)^2 - (s_2 - 2)^2 .$$

(a) [5 points] Find the Nash equilibrium of **G** and show that it is unique.

(b) [8 points] Now consider the infinitely-repeated version of **G** where players have the common discount factor $\delta \in (0,1)$. Fully specify the trigger strategies that will maximize the sum of the equilibrium payoffs of the two players, and solve for the critical discount factor $\hat{\delta}$ such that these trigger strategies form a subgame perfect equilibrium (SPE) whenever $\delta \in (\hat{\delta}, 1)$.

(c) [9 points] Next, fully specify the strategies for the "maximally-collusive equilibrium" that will maximize the sum of the equilibrium payoffs of the two players and which will form an SPE *for the lowest possible discount factor*. (Limit attention to punishments carried out for exactly one period following any deviation.) Again, solve for the critical discount factor $\delta^*$ such that these strategies form a subgame perfect equilibrium (SPE) whenever $\delta \in (\delta^*, 1)$.

(d) [3 points] Are the critical discount factors $\hat{\delta}$ of part (b) and $\delta^*$ of part (c) the same or different? Explain.

4. [25 points] **Two simultaneous auctions**

Consider two **first-price**, sealed-bid auctions that are held simultaneously. Item A is sold in auction A and item B is sold in auction B. Two bidders, subscripted by $i$ = 1,2, participate in each of these auctions. Bidder $i$'s value is $v_i^A$ for item A only and is $v_i^B$ for item B only. Bidder $i$'s value for having both items is $v_i^{AB}$. For each bidder $i$, assume that $v_i^A = v_i^B = v_i$ and $v_i^{AB} = \lambda v_i$, where $v_1$ and $v_2$ are drawn independently from a uniform distribution on [0,1] and $v_i$ is privately known to bidder $i$. The parameter $\lambda \in (1, \infty)$ is common knowledge. Consider two versions of the simultaneous auction game.

(a) In auction game $\mathbf{G^1}$, bidder $i$ ($i = 1,2$) is constrained to submit the same bid in both auctions. That is, bidder $i$ must bid the same amount, denoted by $b_i \in [0, \infty)$, in auctions A and B.

(i) [2 points] What is the set of bidder $i$'s pure strategies in game $\mathbf{G^1}$?

(ii) [8 points] Solve for a Bayesian-Nash equilibrium ***in pure strategies*** of this game.

(iii) [3 points] Is the equilibrium allocation efficient? Your answer should depend on $\lambda$.

(b) In auction game $\mathbf{G^2}$, bidders are allowed to submit different bids in the two auctions.

(i) [2 points] What is the set of bidder $i$'s pure strategies in game $\mathbf{G^2}$?

(ii) [10 points] Is the Bayesian-Nash equilibrium of game $\mathbf{G^1}$ also a Bayesian-Nash equilibrium of game $\mathbf{G^2}$? Your answer should depend on $\lambda$. For full credit, provide a rigorous argument by setting up a bidder's utility maximization problem given that the opponent uses the equilibrium strategy of game $\mathbf{G^1}$. (You do not need to worry about second-order conditions.) For partial credit, provide an intuitive and concise argument in words regarding whether there is a profitable deviation for a bidder.

[Hint: In setting up a bidder's maximization problem, you may want to impose a ranking of the bidder's two bids. For example, without loss of generality, bidder $i$ maximizes over $(b_i^A, b_i^B)$ subject to $b_i^A \leq b_i^B$.]