Economics 603 Professor Lawrence Ausubel
Fall 2020–2021

Final Exam

Answer all four questions.
Write the solutions to different questions on different pages. Write your student number at the beginning of the solution to each question.
Write legibly, show your work, and justify your responses.
Note that there is a different upload location on ELMS for each of Q1, Q2, Q3 and Q4.
Be sure to scan your solutions as pdf files.

1. [25 points] **The Game of Whether or Not to Wear a Mask**

Consider a game with the following story behind it: A long time ago in a galaxy far, far away, there was a pandemic on the planet Tatooine. Each of the N inhabitants of Tatooine was faced with the choice of whether or not to wear a mask. While the masks by themselves were only slightly inconvenient, for unknown reasons, any inhabitant of Tatooine received strong disutility from wearing a mask if they learned that at least two other inhabitants of the planet went without a mask. Meanwhile, masks were only partially effective in protecting the wearer from catching the virus, but they were very effective in preventing the wearer from spreading the virus.

To be more precise, consider the following static game of complete information. The strategy set is {Wear a Mask, Do Not Wear a Mask}. The expected payoff to player i equals:

10 , if player i does not wear a mask and all other players wear masks;

9 , if player i wears a mask and all other players wear masks;

5 , if player i does not wear a mask and exactly one other player does not wear a mask;

4 , if player i wears a mask and exactly one other player does not wear a mask;

$-K^2 - 100$, if player i wears a mask and K other players ($K \geq 2$) do not wear masks; and

$-2K^2$, if player i does not wear a mask and K other players ($K \geq 2$) do not wear masks.

(a) Let $N \geq 25$. Is this game solvable by iterated elimination of dominated strategies? Provide a clear and concise justification for your response.

(b) Let $N \geq 25$. Does this game have a symmetric Nash equilibrium in pure strategies? If so, find it, and if not, provide a clear argument that none exist.

(c) Let $N \geq 25$. Does this game have an asymmetric Nash equilibrium in pure strategies? If so, find ***all*** of them, and if not, provide a clear argument that none exist.

(d) Let $N \geq 25$. Does this game have a Nash equilibrium in strictly mixed strategies? If so, provide a clear argument that one exists—but you are ***not*** being asked to calculate one. If not, provide a clear argument that none exist.

2. [20 points] **A Location Game for Vending Machines**

Consider a town with a continuum of residents, who are distributed uniformly on the interval [0,1]. Two sellers are trying to sell a product by placing some vending machines on the interval. Both sellers sell the same product. Each resident has a unit demand for the product and buys from the nearest vending machine. In the event that there are multiple vending machines nearest to a resident, she randomly selects one of them to make the purchase. The price of the product is fixed at 1. The cost of producing the good is zero. Both sellers seek to maximize their profits.

(a) Seller 1 has **one** vending machine and seller 2 has **two** vending machines.

(i) Specify the strategy set for seller 1 and the strategy set for seller 2.

(ii) Solve for a Nash equilibrium ***in pure strategies*** of this game or prove that there does not exist any Nash equilibrium ***in pure strategies***.

(b) Seller 1 has **two** vending machines and seller 2 has **two** vending machines.

(i) Prove that in any Nash equilibrium, the equilibrium revenue is 0.5 for each seller. [*Hint*: Show that placing one vending machine at 0.25 and the other at 0.75 guarantees seller 1 revenues of at least 0.5 (and similarly for seller 2).]

(ii) Given part (i), argue that the distance between seller 1 or seller 2's two vending machines in any pure strategy Nash equilibrium is exactly 0.5.

(iii) Solve for all of the Nash equilibria ***in pure strategies*** of this game and provide an interpretation of these equilibria.

3. [25 points] **An Infinitely-Repeated Bertrand Game with Three Firms**

Consider an infinitely-repeated Bertrand game with ***three*** firms, each of which has a discount factor of $\delta \in (0,1)$ between periods. In the stage game, the market demand curve is given by $\boldsymbol{Q = 8 - P}$, each firm has a constant marginal (and average) cost of **2**, and prices are restricted to being nonnegative.

(a) In the stage game, determine the symmetric, Pareto-optimal division of the surplus. [Interpret "Pareto optimal" in relation to the three firms only—not in relation to consumers.]

(b) Fully specify a profile of trigger strategies that would provide the payoff division of part (a), and find the critical discount factor δ^* needed for these strategies to form a subgame-perfect equilibrium of the infinitely-repeated game.

(c) Fully specify a profile of "maximally-collusive" strategies that would provide the payoff division of part (a) using ***one-period*** punishments, and find the critical discount factor δ^* needed for these strategies to form a subgame-perfect equilibrium of the infinitely-repeated game. [*Hint*: There are more severe punishments available in this game than the Nash equilibrium strategies, so to be "maximally collusive", you should make use of them.]

(d) Fully specify a profile of "maximally-collusive" strategies that would provide the payoff division of part (a) using ***two-period*** punishments. Write down the system of inequalities that the critical discount factor δ^* must satisfy in order for these strategies to form a subgame-perfect equilibrium—showing which inequalities bind or proving that no such δ^* exists.

4. [30 points] **Sequential All-Pay Auctions**

Two bidders (in part (a)) and three bidders (in part (b)) are competing to win a single item. Bidder i's value for the item is v_i, which is privately known to bidder i. It is common knowledge among the bidders that the v_i are independent and identically distributed on [0,1] according to the cumulative distribution $F(x) = x^{1/2}$.

In the auction, every bidder needs to submit a nonnegative bid and pay his bid regardless of the final outcome. Bidders are risk-neutral and have quasilinear utilities, so if bidder i wins the auction with a bid of b_i, his payoff is $v_i - b_i$; and if he loses, his payoff is $-b_i$.

There are two periods in the auction game. Bidders are assigned to bid in one of the two periods. At the beginning of period 2, the highest bid in period 1 is announced. After observing the highest bid in the previous period, the bidder(s) in period 2 submit their bids. The bidder with the highest bid in the auction wins the item. Ties are settled in favor of period 2 bidders. That is, if a period 1 bidder and a period 2 bidder both have the highest bid, the period 2 bidder wins. Ties between two bidders in the same period are settled by the toss of a fair coin.

This question considers the equilibria in each of the following two specifications of the game.

(a) **Two-bidder all-pay auction: Bidder 1 bids in period 1; and Bidder 2 bids in period 2.**

(i) Suppose that Bidder 1 bids β in period 1. Find Bidder 2's optimal bidding strategy in period 2.

(ii) Given part (i), determine Bidder 1's optimal bidding strategy in period 1.

(b) **Three-bidder all-pay auction: Bidder 1 bids in period 1; and Bidders 2 and 3 bid in period 2.**

(i) Suppose that Bidder 1 bids β in period 1. Determine the critical valuation level $\hat{v}$, such that Bidder 2 will bid at least β in equilibrium in period 2 only if $v_2 \geq \hat{v}$.

(ii) Given part (i), determine Bidder 1's optimal bidding strategy in period 1.

(iii) [Does not depend on part (ii).] Using the derivation of the Revenue Equivalence Theorem, determine the entire optimal bidding strategies of Bidders 2 and 3 in period 2, if Bidder 1 bids β in period 1.