Economics 603 Professor Lawrence Ausubel
Fall 2008–2009

Final Exam

Answer all four questions, each in a *different* blue book.
Write your student number on each blue book and include your name on the book for Question 1.
Please return this question sheet at the same time that you turn in the blue books.
Write legibly, show your work, and justify your responses.

1. [25 points] **The all-pay auction with three allowable bids**

Consider a sealed-bid ***all-pay*** auction for a single object, where there are only ***three*** allowable bids. The two risk-neutral bidders have valuations which are private information and which are drawn from i.i.d. random variables that are uniformly distributed on the interval [0,1]. After observing her own valuation, each of the two bidders simultaneously and independently submits a bid selected from the three-element set $\{0, 1/3, 2/3\}$ (i.e., the only allowable bids are 0, 1/3 and 2/3). The high bidder wins the object, but each bidder (winner or loser) pays the amount of her bid; in the event of a tie, the winner is determined by the toss of a fair coin.

(a) Describe the structure that any Bayesian-Nash equilibrium of this game must take, and provide a full justification why it must take this form.

(b) Solve for the Bayesian-Nash equilibrium of this all-pay auction with three allowable bids.

2. [25 points] **An infinitely-repeated game**

Consider the infinitely-repeated game, with a discount factor of $\delta \in (0,1)$, based on the following stage game:

		Player 2		
		L	C	R
Player 1	U	10, 10	2, 11	0, 13
	M	11, 2	6, 7	0, –1
	B	12, –2	0, –5	3, 3

(a) Find all pure-strategy Nash equilibria of the stage game.

(b) Fully specify a pair of pure strategies in the infinitely-repeated game which, if followed, yield (10,10) as the average payoff.

(c) Find the critical discount factor to support the strategies of part (b) as a subgame-perfect equilibrium of the infinitely-repeated game.

(d) Is it possible to support (10,10) as an average payoff using a trigger-strategy equilibrium ***with a pure-strategy punishment*** when $\delta = 0.275$? If it is possible, carefully specify the equilibrium strategies. If it is impossible, briefly explain why.

(e) Is it possible to support (10,10) as an average payoff using a trigger-strategy equilibrium ***with a mixed-strategy punishment*** when $\delta = 0.275$? If it is possible, carefully specify the equilibrium strategies. If it is impossible, briefly explain why.

3. [25 points] **Cournot oligopoly with fixed costs**

There are four symmetric firms in a market. The inverse demand function for this market is given by $P(Q) = 11 - Q$, where $Q = \sum_{i=1}^{4} q_i$ is total quantity. Each firm *i* simultaneously and independently chooses a quantity q_i (i.e. as in a standard Cournot model), but each firm has the following cost function: $C(q) = \begin{cases} 0 & \text{, if } q = 0 \\ q + F & \text{, if } q > 0 \end{cases}$, where *F* is a parameter of the model.

(a) Argue why is it possible to have equilibria with different number of firms which produce at positive level of output for the same values of *F*.

(b) Solve for all pure-strategy Nash equilibria, for all possible $F \in [2, 5]$.

(c) Consider a two-stage dynamic game where in the first stage firms decide whether or not to enter, and in the second stage (after observing the entry decision of each competitor) firms compete in quantities. If the firm decides to enter, it must pay *F*. Following entry, the cost function of production is $C(q) = q$. For all $F \in [2, 5]$, find all SPE in pure strategies. [Hint: Use the equilibria of part (b) and argue which of them are SPE.]

(d) Consider the dynamic game of part (c) with the following modification. After making the entry decision, the firms ***do not observe*** the entry decision of their opponents. Explain how to analyze this game and, for all $F \in [2, 5]$, find all SPE in pure strategies.

4. [25 points] **The "median bid" auction model**

Consider the following auction game which, somewhat surprisingly, was proposed recently for a real-world application. There are *n* risk-neutral bidders for a single object. Each bidder *i* has a value $v_i \in [0,1]$ for an object and each bidder *i* is required simultaneously and independently to submit a bid $b_i \in [0,1]$ for the object. When *n* is an odd number, the bidder who submitted the median bid (i.e. the bid ranked ½ (*n* + 1) when the bids are ranked from highest to lowest) is declared the winner and pays the amount of its bid. When *n* is an even number, the bidder who submitted the bid immediately above the median (i.e. the bid ranked ½ *n* when the bids are ranked from highest to lowest) is declared the winner and pays the amount of its bid. Ties among multiple winning bids are resolved randomly, applying an equal probability of winning to each bid.

(a) Suppose that there are three bidders (i.e. $n = 3$) and this is a game of ***complete*** information. In particular, assume that the value v_i of each bidder *i* is commonly known and that $v_1 > v_2 > v_3$. Determine ***all*** of the pure-strategy Nash equilibria of this game.

(b) Suppose that there are four bidders (i.e. $n = 4$) and this is a game of ***complete*** information. In particular, assume that the value v_i of each bidder *i* is commonly known and that $v_1 > v_2 > v_3 > v_4$. Determine ***all*** of the pure-strategy Nash equilibria of this game.

(c) Suppose that there are two bidders (i.e. $n = 2$) and this is a game of ***incomplete*** information. In particular, assume that the value v_i of each bidder *i* is uniformly distributed on the interval [0,1] and is privately known by bidder *i* only. Determine ***all*** of the Bayesian-Nash equilibria of this game.

(d) Suppose that there are three bidders (i.e. $n = 3$) and this is a game of ***incomplete*** information. In particular, assume that the value v_i of each bidder *i* is uniformly distributed on the interval [0,1] and is privately known by bidder *i* only. Determine ***all*** of the Bayesian-Nash equilibria of this game.