Time Inconsistency in the Credit Card Market

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January 30, 2005

Abstract

This paper analyzes a unique dataset, which contains results of a large-scale experiment in the credit card market. Two puzzling phenomena that suggest time inconsistency in consumer behavior are observed. First, more consumers accept an introductory offer which has a lower interest rate with a shorter duration than a higher interest rate with a longer duration. However *ex post* borrowing behavior reveals that the longer duration offer is a better offer, because respondents continue borrowing on the credit card. Second, consumers are reluctant to switch, and many of those consumers, who have switched before, fail to switch again later. Our study shows that standard exponential preferences cannot explain the observed behavior because they are time consistent. However hyperbolic preferences which are time inconsistent come closer to rationalizing the observed behavior. In particular, two special cases of hyperbolic discounting are carefully examined, sophisticated and naive. Sophisticated consumers prefer the short offer because it serves as a self-commitment device. Naive consumers prefer the short offer because they underestimate their future debt. Estimation results based on a realistic dynamic model suggest that consumers have a severe self-control problem, with a present-bias factor $\beta = 0.8$, and that the average switching cost is \$150. With the estimated parameters, the dynamic model can replicate quantitative features of the data.

1 Introduction

Does consumer behavior exhibit time inconsistency? This is an essential, yet difficult question to answer. Since the pioneering contribution of Samuelson (1937), it has become a standard assumption in dynamic economics models that consumers have an exponential time discount function, $\{1, \delta, \delta^2, ...\}$, which implies that consumer behavior is *time consistent*. A significant body of evidence in experimental psychology and economics literature, however, suggests that consumers discount the future hyperbolically, not exponentially. The essential feature of hyperbolic discounting is that consumers are *time inconsistent*. In the last decade, a particular kind of hyperbolic discounting, the quasi-hyperbolic discount function, $\{1, \beta\delta, \beta\delta^2, ...\}$, has been widely studied due to its analytic simplicity.[1] Many researchers have applied this discount function to explain various economic anomalies, such as procrastination, retirement, addiction and credit card borrowing.[2] This paper also adopts this formulation, which shall be simply referred to as hyperbolic discounting in later discussion.

The recent use of hyperbolic discounting has been criticized for lack of convincing empirical evidence.[3] An ideal test is to compare consumers' long-run plans with their later actions, which will be consistent for exponential consumers but inconsistent for hyperbolic consumers. In the real world, it is difficult to track long-run plans or later actions — especially long-run plans.

This paper examines time inconsistency using a large-scale randomized experiment in the credit card market, with which we have a unique opportunity to conduct a reasonably good test. In the experiment, 600,000 consumers were each randomly assigned to one of six different groups, denoted as Market Cells A to F, which were mailed six different credit card offers. The six offers had different introductory interest rates and different durations: Market Cell A (4.9% for 6 months), B (5.9% for 6 months), C (6.9% for 6 months), D (7.9% for 6 months), E (6.9% for 9 months) and F (7.9% for 12 months). All other characteristics of the solicitations were identical across the six market cells.

[1] The quasi-hyperbolic discounting accomodates three different hyperbolic time preferences as special cases: naive, sophisticated and partial naivete. We will discuss their difference in more details later. Naive and sophisticated hyperbolic discounting are commonly applied in theoretic studies.

[2] See Akerlof (1991), Diamond and Koszegi (2003), Gruber and Koszegi (2001), Harris and Laibson (2001), Laibson (1997), Laibson *et al.* (1998), O'Donoghue and Rabin (1999, 2001).

[3] For example, Mulligan (1997), Fernandez-Villaverde and Mukherji (2002), Rubinstein (2003) and Besharov and Coffey (2003).

Consumer responses and subsequent usage of respondents for 24 months were observed.

One advantage of this experiment is that the 600,000 subjects do not change their behavior due to their participation in the experiment; indeed, they do not even know that they are part of an experiment. A second advantage of this experiment is that consumer long-run plans can be inferred from their actions. Consumer plans are identified from their responses to different offers, such as A (4.9% for 6 months) and F (7.9% for 12 months). For example, if the consumers who receive the short introductory offer (A) are more likely to accept the credit card than those who receive the longer introductory offer (F) — and, given the randomized experimental treatment, the two groups may be viewed as identical — it implies that the consumers expect their credit card debt to be short-lived. For purposes of inferring experimental subjects' long-run plans, actions speak much louder than words. A third advantage of this experiment is that the number of experimental subjects (600,000 consumers solicited, and more than 5,000 consumers accepting the solicitation) is quite large, ensuring that the inferences drawn will be precise. Combining the subjects' inferred plans with their later actions, we have a unique opportunity to test for time consistency.

There are two phenomena in this dataset suggestive of time inconsistency. First, significantly more consumers in Market Cell A are found to accept their offers than in Market Cell F. This *ex ante* preference becomes puzzling after observing that respondents, *ex post*, keep on borrowing on this card well after introductory periods. We will show in a later section that respondents in Market Cell A would pay less interest if their cards were repriced as offer F. Why do not all their counterparts in Market Cell F accept the F offer? We term the first puzzle as "rank reversal." Second, consumer switching behavior is not consistent over time. The majority of respondents (60%) stay with this card after the introductory period, and their debts remain at the same level as when they accepted this card. Given the same debt level, it should be worthwhile to switch a second time since it was optimal to accept this offer before. Obviously, there would be no puzzle if respondents did not receive new low-rate solicitations from other credit card issuers after the end of the introductory period. However, the number of solicitations averaged at least three per qualified household per month during the sample period. A typical solicitation from the observed issuer (and other credit card issuers contemporaneously) included a 5.9% introductory interest rate for 6 months. 96% of the respondents remain credit-worthy after 6 months, which will be discussed in greater detail in section 3.

At least two explanations are possible for consumer behavior that on the surface appears to be time inconsistent. First, consumers may behave in a time inconsistent fashion because they have hyperbolic time preferences. Hyperbolic consumers have a much higher discount rate in the short run than in the long run. Therefore their credit card choice, which is largely determined by short-run benefit, may not be optimal from the long-run perspective. Second, consumers are subject to random shocks, the *ex post* realizations of which may generate divergences between consumers' initial plans and later actions, even if their preferences are time consistent.

In this paper, we examine the validity of both hypotheses. To build up a basic intuition, we analyze a multi-period credit card choice model without uncertainty. The simple model shows that exponential consumers will never exhibit "rank reversal". Exponential agents always prefer an offer requiring less interest payment. This is due to their time consistency, which makes their short-run choice (credit card choice) also optimal from the long-run perspective (later interest payment). However, "rank reversal" is possible for hyperbolic consumers. There are two kinds of hyperbolic preferences which have been widely studied in the literature: *sophisticated* and *naive*. Our studies show that both versions are able to explain "rank reversal", even though the underlying economic stories are different. A sophisticated hyperbolic consumer who recognizes her time inconsistency problem would like to precommit to avoid overspending in the future. Accepting a shorter introductory offer, rather than a longer one, serves as a commitment device, even though she would pay less interest if she accepted the longer offer. A naive hyperbolic consumer, however, trades a longer offer for a shorter one because she underestimates the amount she will borrow in the future. This underestimation is due to the fact that she naively believes that her future selves will be as patient as she desires now.

To explore the possibility of explaining behavior with random shocks, we develop a dynamic model which incorporates three important random processes. First, consumer income has both persistent and transitory shocks. Second, receiving new introductory offers is probabilistic. Third, accepting a new offer causes the consumer to incur a random switching cost. A realistic dynamic model is required because some researchers argue that exponential discounting can explain anomalies if "even a small degree of" uncertainty is incorporated,For example Fernandez-Villaverde (2002). which we show is not necessarily the case here.

We find that an exponential model still cannot reconcile respondents' continued borrowing and preference for the shorter offer A, even with random shocks. The intuition for the failure is that the behavioral discrepancy observed is not for some individuals but for a large group of consumers. An individual exponential consumer may, *ex ante*, accept an offer that proves, *ex post*, to be a bad deal based on the realized random shocks. However, a relatively large group of exponential consumers should prefer the offer that *on average* provides the lowest interest payment. Hyperbolic time preferences are also incorporated into the dynamic model, from which we estimate time preference parameters with a reasonable degree of precision.

Estimation results show that the second puzzle can only be explained by the stochastic nature of switching costs, which are traditionally assumed to be constant for an individual. Our random switching cost appears to be a more realistic treatment, because it captures either fluctuations in free time or fluctuations due to subjective, psychological factors that strongly affect realized switching costs. Under this interpretation, respondents in this experiment accept the offers due to their low realized switching costs at the time of solicitation. However, their mean switching costs are much higher, which can be partially inferred from the low response rate (1%). This high mean will keep the majority of respondents from switching a second time after the introductory period.

The paper is organized as follows. There have been many empirical studies in support of hyperbolic discounting, both from laboratory experiments and field studies, which will be discussed in detail in the following section. In Section 3, the experiment is introduced and the two puzzles are elaborated. Section 4 rigorously defines what we call "rank reversal" and proves that it is impossible in an exponential model with certainty. A simple 3-period model illustrates that "rank reversal" is possible for hyperbolic agents. The dynamic model with uncertainty, which accommodates both exponential and hyperbolic time preferences, is presented in Section 5. The estimation strategy and results are discussed in Section 6. Section 7 concludes.

2 Related Empirical Studies of Hyperbolic Discounting

The most cited empirical evidence on hyperbolic discounting is from laboratory experiments.[4] One major problem with laboratory evidence is that most experiments only elicit consumer time preferences once. In Ainslie and Haendel (1983), for example, experimental subjects are asked the

[4]See Ainslie and Haendel (1983), Loewenstein and Thaler (1989) and Thaler (1981).

following two questions:

> Question 1: Would you rather receive \$50 today or \$100 in 6 months?
>
> Question 2: Would you rather receive \$50 in one year or \$100 in 1 year plus 6 months?

Many subjects choose the smaller-sooner reward in the first question and the larger-later reward in the second. This phenomenon has been termed as "preference reversal" and is cited as empirical evidence against exponential discounting. The argument is that subjects apparently apply a larger discount rate for a six-month delay as the delay becomes closer, while exponential time preferences assume that consumers use the same discount rate for any equal-distance period. However, this "preference reversal" can also be explained by risk aversion and uncertainty. Risk averse consumers prefer \$50 today to \$100 in 6 months because the first offer has no uncertainty. However, both offers in Question 2 involve uncertainty, therefore the large dollar value offer is better.[5]

The essential difference between an exponential and hyperbolic consumer concerns whether the "current self" and "future self" agree on the desired discount factor in the future, not whether the discount factor is exactly the same for any two time periods of equal length. An exponential consumer applies the same discount factor (δ) between period t and $t+1$ no matter which period it is currently. However, a hyperbolic consumer applies a discount factor δ between period t and $t+1$ at periods $\tau < t$, and a discount factor $\beta\delta$ at period t. Because of this, a hyperbolic consumer would like to revise her consumption plan for period t when period t arrives. This revision does not exist in the exponential model. Therefore, to identify hyperbolic discounting, it is vital to solicit consumer time preferences in multiple periods.

Several multi-period experiments have been conducted, such as Read and van Leeuwen (1998), in which subjects were asked to choose between healthy and unhealthy food both in advance and immediately before the snacks were given. They found that subjects were more likely to make the unhealthy choice when asked immediately before the snacks were to be given than when asked a week in advance. However, this evidence is also questionable, subjects may not tell the truth when they were first asked, because they knew they could always change their minds later.

Since eliciting consumer true time preferences from laboratories is difficult, some researchers have attempted to infer consumer time preferences from their economic behavior in the real world.

[5] See Fernandez-Villaverde (2002) for more details.

Researchers have analyzed consumer behavior in different markets, such as the credit card market (Laibson, Repetto and Tobacman, 2000, 2004), the health club market (Della Vigna and Malmendier, 2003), and the labor market (Fang and Silverman, 2001, Paserman, 2001).

Among these studies, the closest relative to our work is Della Vigna and Malmendier (2003), which utilizes a similar identification strategy. Consumer time inconsistency is identified by comparing initial contractual choices with subsequent day-to-day attendance. They find that health club members who sign a monthly contract would be better off if they chose to pay per visit. The disadvantage of that study is that they focus on first-time users. Inexperienced users may choose the wrong contract because they have incorrect expectations about their future attendance. Actually they find strong evidence that club members learn over time: they switch to a more appropriate contract given their actual attendance. An experienced sample is very important when identifying consumer time-inconsistency from behavior at two different dates. In the next data section we will show that consumers in our sample are very familiar with credit card offers.

3 A Unique Dataset

A substantial portion of credit card marketing today is done via direct-mailed pre-approved solicitations. The typical solicitation includes a low introductory interest rate for a known duration, followed by a much higher post-introductory interest rate. Sophisticated card issuers decide on the terms of their solicitations by conducting large-scale randomized trials. The dataset used is the result of such a "market experiment" conducted by a major United States issuer of credit cards in 1995. The issuer generated a mailing list of 600,000 consumers and randomly assigned the consumers into six equal-sized market cells (A-F). The market cells have different introductory offers as mentioned above but are otherwise identical (for example the same post-introductory interest rate of about 16%).

In each market cell, between 99860 and 99890 observations are actually obtained, out of the 100,000 consumers. About half of the missing observations are due to one known data problem: approximately 5% of the individuals who respond to the pre-approved solicitation but are declined (due to a deterioration of credit condition or failure to report adequate information of income) are deleted from the dataset for unknown reasons. Nevertheless, over 99.8% of the sample is still included. Ausubel (1999) offers statistical evidence that this is still a good random experiment among the remaining observations. Credit bureau information of the 599,257 consumers are observed at the time of solicitation and their responses to the offers are recorded.

For consumers who accept their credit offers ("respondents"), we observe detailed information about their monthly account activities for subsequent 24 months. For a month t, we observe the amount paid on the account during the month, the amount of new charges during the month, any finance charge (such as interest, late-payment fee and over-credit-limit fee) and the total balance owed at the end of the month. Based on the information, we distinguish credit card debt from convenience charges, for which no interest is billed. In later analysis, we will focus on debt.

Besides these quantitative statistics, we also observe two interesting qualitative statistics. The first measures the delinquency status: whether the account is delinquent this month or not and the duration of the delinquency. The second measures whether the account owner has filed for a personal bankruptcy or not. These two measures offer important information about the respondent's credit status over time.

Important financial statistics for the whole sample and for respondents are reported in Table (**??**). Most observables of respondents are statistically worse than the whole sample. Nevertheless, both groups are established good credit risk. Majority of consumers have more than a ten-year credit history. Every consumer has at least one existing credit card and 75% have more than two credit cards. They are good risk because very few have been past due in last two years, which is shown in "Number of Past-due". And none of them has had a sixty-day past-due, which is considered to be a severe delinquency. Their good credit quality can also be inferred from their high revolving limit and low revolving balance. "Revolving Limit" is the total credit limit a consumer has on her revolving accounts.[6] "Revolving Balance" is the total balance on those revolving accounts, including both convenience charges and credit card debt. For a better description, a *utilization rate* is introduced, defined as the ratio of revolving balance to its limit. The average utilization rate for the whole sample is only 16% and for respondents only 27%. Good credit risk generally receive many solicitations every month, especially when they have a long credit history.

There are two puzzling phenomena observed in this dataset. The first puzzle is that significantly more consumers in Market Cell A (4.9% for 6 months) accept their offers than in Market Cell F

[6]Revolving accounts are the accounts on which consumers can borrow with no prespecified repayment plan. The majority of revolving accounts are credit cards.

(7.9% for 12 months). However, respondents keep on borrowing on this card after six months. Based on *ex post* interest payment offer F should have a higher response rate than offer A. This phenomenon is called "rank reversal". Consumer responses are recorded in the third column of Table (**??**). Only about one percent consumers accept their credit card offers, which is also the average response rate for the whole economy in the sample period.[7] Significantly more consumers accept the shorter offer A than the longer offers, E (6.9% for 12 months) and F. This preference is suboptimal if one compares the effective interest rates under different offers. The effective interest rate is the annual interest rate respondents actually pay in each market cell, which equals the ratio of the total interest payment to the total credit card debt and is shown in the fifth column of Table (**??**). The effective interest rate is two percentage points lower in Market Cell F than in Market Cell A and one percentage point lower in Market Cell E. Since the average debt among borrowers is $2500, an average borrower in Market Cell A pays $50 more interest than in Market Cell F and $25 than in Market Cell E. To make sure this "rank reversal" phenomenon is not driven by outliers, we calculate a "what if" interest payment for each respondent. We ask how much more or less a member of Market Cell A would pay if her account were repriced according to the formula of Market Cell F. Consumer behavior is assumed unchanged under the new cell. 42% of them would save more than $10, 34% would save more than $20 and 26% would save more than $40. Only 21% of respondents would do worse in this exercise. One thing deserves mentioning is that consumers optimally prefer the lower introductory interest rates among offers A, B(5.9% for 6 months), C(6.9% for 6 months) and D(7.9% for 6 months). This seems to suggest consumers are rational and they can make the right choice when comparison involved is simple. This also makes us more confident about the quality of this random experiment.

The second puzzle is that respondents do not switch again after the introductory offer expires even though their debts remain at the same level as before. We observe a stable debt distribution over time among respondents who borrow. The median debt among borrowers stabilizes around $2000 in the twenty-four months, shown in Fig.(**??**). The first quartile remains around $3500 and the third quartile is around $500. The proportion of respondents who borrow does not decrease much over time. As shown in Fig.(**??**), about 60% of respondents borrow during introductory periods and over 35% continues to carry balances after two years, which is the same across all market cells.

[7] According to BAI Global Inc., the response rate to solicitations is 1.4% in 1995.

Majority of revolvers don't switch. Of course, this is not a puzzle if respondents have not received new offers after this one expires, however, this is impossible given the high volume of solicitations and the good credit quality of respondents. Credit card companies will not send a consumer new solicitations if she is either more than 60 days past-due or she declares a personal bankruptcy. Among respondents, about 1% declare bankruptcy and 4% experience a severe delinquency after accepting this card. Apparently, this cannot explain why 35% respondents don't switch.

4 A Multi-Period Model without Uncertainty

In this section, we will analyze a multi-period model with certainty to prove that "Rank Reversal" is impossible in an exponential model. Time consistent agents will always choose a credit offer which provides the lowest interest payment. However, this possibility exists for hyperbolic agents, both naive and sophisticated, which is illustrated by a simple three-period model. Regardless of its simplicity, the three-period model illustrates essential differences between exponential and hyperbolic models.

Besharov and Coffey (2003) concluded that hyperbolic time preferences are not identifiable using financial rewards. The financial reward they considered is a specific type: giving a certain amount of money to agents at different dates, as is commonly observed in laboratory experiments. The below model provides a specific example where hyperbolic discounting is identifiable, if the financial rewards are carefully designed. Our later estimation work, which is based on a realistic dynamic model, shows that this identification still holds when uncertainty and liquidity constraints are incorporated.

4.1 Time Preferences and Model Set-up

A general time preference formulation[8] is adopted, which incorporates exponential and hyperbolic time preferences. The representative agent has a current discount function of $\{1, \beta_0\delta, \beta_0\delta^2, ...\}$, where β_0 represents "a bias for the present", i.e. how much the agent favors this period versus later periods and δ is a long-term discount factor. The expected future discount function is assumed to be $\{1, \beta_1\delta, \beta_1\delta^2, ...\}$ for all subsequent periods. β_1 is the present bias factor in the future.

[8] This formulation is first developed in O'Donoghue and Rabin(2001).

Depending on the magnitude of β_0 and β_1, this formulation represents four kinds of time preferences, standard exponential discounting and three kinds of hyperbolic discounting. When $\beta_0 = \beta_1 = 1$, this is the standard exponential discounting. Exponential agents have no special preference for current and discount any two consecutive periods by the same discount factor δ. When $\beta_0 = \beta_1 = \beta < 1$, the agent (sophisticated hyperbolic) has a correct expectation about her future. She realizes that the discount factor between period t and period $t+1$ will become $\beta\delta$ when period t arrives, however δ is desired in earlier periods. When $\beta_0 < \beta_1 = 1$, the agent is called a naive hyperbolic agent since she has an incorrect expectation about her future. She naively believes that she would behave herself ($\beta_1 = 1$) from next period on. In between sophisticated and naive hyperbolic agents, a partial naive agent can be defined when $0 < \beta_0 < \beta_1 < 1$. Such an agent underestimates the impatience she has in later periods like a naive agent. However, she anticipates a difference between today's desired patience and tomorrow's actual patience. In the following discussion, we will focus on the first two types of hyperbolic models[9].

The representative agent lives for T periods. At the beginning of period τ, she chooses an optimal consumption level by maximizing a weighted sum of her utilities from this period on:

$$\max_{C_\tau} u(C_\tau) + \beta_0 \sum_{t=\tau+1}^{T} \delta^{t-\tau} u(C_t), \tag{1}$$

where the relative weights are determined by her *current* discount function and where $u(\bullet)$ is a concave instantaneous utility function.

The agent receives an income y_t at period t and she can borrow or save (A_t) at the save gross interest rate r_t, with no limit.[10]

$$C_t = y_t - A_t + r_{t-1}A_{t-1} \tag{2}$$

She has an initial debt A_0 at the beginning of period one. The boundary condition is that she pays

[9]Naive and sophisticated hyperbolic models have been widely studied. Strotz (1956) and Phelps and Pollak (1968) carefully distinguish the two assumptions, and O'Donoghue and Rabin (1999) studies different theoretic implications from these two. Laibson (1994, 1996, 1997) assume consumers are sophisticated. On the other hard, Akerlof (1991) adoptes the naive hyperbolic assumption.

[10]The assumption of frictionless financial markets will be relaxed in the later dynamic model, in which the agent faces credit limit and the borrowing and saving interest rates are not equal.

off all her debt in the last period, i.e. $A_T = 0$. The interest rates $\{r_t\}_{t=1}^T$ are determined by her credit card choice in the first period.

4.2 "Rank Reversal"

As discussed in section 3, "rank reversal" seems to suggest that consumers are time inconsistent. Here we will use the above model to demonstrate that the consistent exponential model indeed cannot explain "rank reversal". Only hyperbolic models, where agents are time inconsistent, can rationalize this behavior.

Suppose there are two introductory offers L (r_L, Γ_L) and M (r_M, Γ_M) in the first period, where r_i and Γ_i are the introductory interest rate and duration respectively and $i \in \{L, M\}$, assuming $r_L < r_L$ and $\Gamma_M < \Gamma_M$. Offer L provides a lower introductory interest rate, however, for fewer periods. The representative agent ranks the two credit offers, based on the optimal utility she would receive under each card offer. To simplify the model, there are no more new offers in later periods.

Definition 1: *"Rank Reversal" occurs if the optimal utility under offer L is larger than under M, however the agent would have paid less interest or received more interest income under M, assuming the asset choice under L.*

Mathematically paying less interest can be formulated as

$$PDV_{L,M}\left(\left\{A_t^L\right\}_{t=1}^T\right) < PDV_{M,M}\left(\left\{A_t^L\right\}_{t=1}^T\right),$$

where $\left\{A_t^L\right\}_{t=1}^T$ denotes the optimal debt path under offer L and

$$PDV_{j,i}\left(\left\{A_t^L\right\}_{t=1}^T\right) = \sum_{t=1}^T \frac{A_t^L\left(r_t^j - 1\right)}{\prod_{s=1}^t r_s^i}$$

is the present discounted value of corresponding interest income, where $i, j \in \{L, M\}$.

The "Rank Reversal" essentially means that the agent's preference order in the utility space is different from that in the financial payment space. She prefers the short offer even though she would have paid less interest (received more interest income) for the same debt (asset) path with the longer offer.

Furthermore, if the agent pays less interest under M, the consumption path $\left\{C_t^L\right\}_{t=1}^T$ is also financially feasible under offer M, as shown the Lemma 1.

Lemma 1:

$$PDV_{L,M}\left(\left\{A_t^L\right\}_{t=1}^T\right) < PDV_{M,M}\left(\left\{A_t^L\right\}_{t=1}^T\right)$$
$$\Rightarrow \sum_{t=1}^{T} \frac{C_t^L}{\prod_{s=1}^{t-1} r_s^M} < \sum_{t=1}^{T} \frac{C_t^M}{\prod_{s=1}^{t-1} r_s^M} = \sum_{t=1}^{T} \frac{y_t}{\prod_{s=1}^{t-1} r_s^M} + A_0.$$

We will use Lemma 1 to prove an important proposition. Proof of Lemma 1 is straightforward, applying Eq. (**??**).

4.2.1 "Rank Reversal" Impossible for Exponential Agents

Before prove the proposition, we will first layout two definitions and prove one Lemma.

Definition 2: *A game with commitment is one in which self 1 chooses an optimal consumption plan according to her preference, and all later selves are required to follow the plan. Self 1's problem, given an offer i, is the following:*

$$\begin{array}{ll} & \max_{\{C_\tau\}_{\tau=1}^T} u(C_1) + \beta_0 \sum_{t=2}^T \delta^{t-2} u(C_t) \\ s.t. & \sum_{t=1}^T \frac{C_t}{\prod_{s=1}^{t-1} r_s^i} = \sum_{t=1}^T \frac{y_t}{\prod_{s=1}^{t-1} r_s^i} \end{array}$$

Definition 3: *A game without commitment is one in which self τ chooses her optimal consumption given the initial asset, $A_{\tau-1}$, and she has no control over future selves' choices. The only way she may influence future behavior is by changing the state variable A_τ. The problem is defined as:*

$$\begin{array}{ll} V_\tau(A_{\tau-1}) & = \max_{C_\tau} u(C_\tau) + \beta_0 \delta \widetilde{v}_{\tau+1}(A_\tau) \\ s.t. & C_\tau = y_\tau - A_\tau + r_{\tau-1} A_{\tau-1} \end{array}$$

where $\widetilde{v}_{\tau+1}(A_\tau)$ is the expected continuation utility.

The essential difference between the two games is their choice sets. The choice set for the game without commitment is only a subset of that for the game with commitment. The budget constraint is the only constraint in the game with commitment. However, the game without commitment has an additional constraint which is her future behavior. Some financially feasible plan may not be her choice because its implementation issue.

Lemma 2: *For an exponential model, solutions are the same for the game with or without commitment.*

The lemma is true due to the Principle of Optimality, Bellman (1957).

Proposition: *Exponential agents will never exhibit " Rank Reversal".*

Proof: Consumers' credit card usage is best described as a game without commitment defined in Definition 3. For exponential agents, the choice is also optimal for the game with commitment given Lemma 2. Hence the asset path should provide the highest utility among all financially feasible plans. If $PDV_{L,M}\left(\left\{A_t^L\right\}_{t=1}^T\right) < PDV_{M,M}\left(\left\{A_t^L\right\}_{t=1}^T\right)$, the optimal consumption path under L is also feasible under M as shown in Lemma 1. The optimal choice under M should be better than that under offer L, i.e. offer M should have been chosen instead of L. Therefore, it cannot be the case that $PDV_{L,M}\left(\left\{A_t^L\right\}_{t=1}^T\right) < PDV_{M,M}\left(\left\{A_t^L\right\}_{t=1}^T\right)$ while offer L is preferred to M.

4.2.2 "Rank Reversal" Possible for Hyperbolic Agents

However, "Rank Reversal" is possible in hyperbolic models. The key reason is that hyperbolic time preference is not consistent. Some consumption plans are not optimal in the future even though they are both financially feasible and preferred in the first period. Therefore, it is possible that the chosen consumption plan which is optimal in every period may incur higher costs than those plans.

We analytically solve the above model, where $T = 3$ and $u(C_t) = C_t^{1-\rho}/(1-\rho)$. The optimal asset decision is the following:

$$\begin{aligned}
A_1 &= \frac{(y_1 + A_0) - Z\left(y_3 + r_2 y_2\right)}{1 + r_1 r_2 Z}, \\
A_2^{exp} &= \frac{y_2 + r_1 A_1 - \left(\beta_1 \delta r_2\right)^{-\frac{1}{\rho}} y_3}{1 + \left(\beta_1 \delta r_2\right)^{-\frac{1}{\rho}} r_2}, \\
A_2^{actual} &= \frac{y_2 + r_1 A_1 - \left(\beta_0 \delta r_2\right)^{-\frac{1}{\rho}} y_3}{1 + \left(\beta_0 \delta r_2\right)^{-\frac{1}{\rho}} r_2},
\end{aligned}$$

in which

$$Z = \left[\beta_0 \delta r_1 r_2 \left(\frac{X}{1 + X r_2}\right)^{1-\rho} + \beta_0 \delta^2 r_1 r_2 \left(\frac{1}{1 + X r_2}\right)^{1-\rho}\right]^{-1/\rho},$$

where $X = \left(\beta_1 \delta r_2\right)^{-\frac{1}{\rho}}$. A_2^{exp} is the expected behavior of self 2 from self 1's point of view and A_2^{actual} is actual behavior of self 2. The two are the same in the sophisticated hyperbolic model, where $\beta_0 = \beta_1$.

Self 1 underestimates (overestimates) her debt (saving) at period 1 when $\beta_0 < \beta_1$. Given A_1, $A_2^{actual} \leq A_2^{exp}$, since $dF/d\beta > 0$ [11] and $\beta_1 \geq \beta_0$, where

$$F = \frac{y_2 + r_1 A_1 - (\beta\delta r_2)^{-\frac{1}{\rho}} y_3}{1 + (\beta\delta r_2)^{-\frac{1}{\rho}} r_2}.$$

Given all other parameters, does a naive hyperbolic agent borrow more than a sophisticated agent in the first period? Intuitively the naive agent should borrow more since she doesn't expect herself to borrow so much in the second period. On the other hand, the sophisticated consumer should accommodate future overspending by borrowing less in the first period. Actually the answer depends on ρ. When $\rho < 1$, $dA_1/d\beta_1 > 0$, i.e. the naive consumer saves more. When $\rho > 1$, $dA_1/d\beta_1 < 0$, i.e. the naive consumer borrows more. When $\rho = 1$, they behave the same. The intuition is that when $\rho > 1$ the agent really would like to smooth consumption over time. Therefore a sophisticated self 1 would like to borrow less to accommodate her borrowing in the second period. However when $\rho < 1$ the self 1 doesn't care much about smoothing consumption. If she knows that self 2 will spend too much, she would leave less wealth to self 2, i.e. borrowing more. The mathematical proof is at the Appendix.

Another interesting finding from the above analytic solutions is that the naive model is similar to the sophisticated model when the present bias is small ($\beta_0 \to 1$). The difference between the two models explodes when $\beta_0 \to 0$. Given A_1, both naive and sophisticated agents will borrow (save) according to A_2^{actual}. The difference in A_1 is from Z. $dZ/d\beta_1$ is a function of $(\beta_0)^{-1/\rho}$, as shown in the appendix. When $\beta_0 \to 0$, $(\beta_0)^{-1/\rho} \to \infty$, since $\rho > 0$. Therefore $dZ/d\beta_1 \to \infty$ and $dA_1/d\beta_1 \to \infty$. On the other hand, when $\beta_0 \to 1$, so does β_1 since $\beta_1 \geq \beta_0$. $dZ/d\beta_1$ is a function of $(\beta_1 - 1)$. Therefore $dZ/d\beta_1 \to 0$ as $(\beta_1 - 1) \to 0$.

We will use numerical examples to illustrate some other interesting findings, which are not easy to see from the analytic solution. Assume $\rho = 2$, $y_1 = y_2 = y_3 = 1$ and $A_0 = 0$. Offer L carries an interest rate of 5% for the first period and 20% for the second period. Offer B has a flat interest rate schedule: 10% for both periods. Fig.(**??**) plots the rank reversal region (the shaded area) in β and δ space, for sophisticated and naive models. For both models there are only two preference parameters. For the sophisticated agent $\beta_1 = \beta_0 = \beta$. The naive agent has a $\beta_1 = 1$,

[11] All derivatives are evaluated in Appendix.

$\beta_0 = \beta$Apparently, there is no rank reversal when $\beta = 1$, which is the exponential model. However, there exists a wide rank reversal area for hyperbolic models.

A naive agent exhibits "rank reversal" because she underestimates her future borrowing. For example, suppose $\beta = 0.82$ and $\delta = 1$. In the first period, she prefers offer A because she expects to save in the second period, $A_2^{\exp} = 0.0312$. However, when the second period arrives she gives in to her instantaneous desire and borrows again, $A_2^{actual} = -0.0135$. Base on her actual behavior, she has made a suboptimal choice in the first period. However, her decision is optimal based on her expectation.

A sophisticated agent does not behave suboptimally because of incorrect expectations, rather because she tries to align her future behavior with her current preference. Continue to suppose $\beta = 0.82$ and $\delta = 1$. If she can commit to her future behavior, she will choose $A_1 = -0.0207$ and $A_2 = 0.0312$. However, she anticipates that this plan will not be followed in the second period. Still she decides to borrow less in period one ($A_1 = -0.0199$) to accommodate tomorrow's borrowing ($A_2 = -0.0131$). Based on her reduced first-period debt, the interest payment under L is more than M. However it is not optimal to choose M since this consumption plan will not be implementable if M is chosen. Given $A_1 = -0.0199$, she will borrow much more under offer M in the second period ($A_2 = -0.0347$), which is worse from her first period's point of view.

Given any δ, a smaller β makes rank reversal more likely. The intuition is as the difference between the long term desired discount factor (δ) and the short term temptation (β) becomes larger, an naive agent's underestimation error is larger and the sophisticated agent is more desperate to constrain herself. Both will lead to financially suboptimal behavior.

Only when β is very small, less than 0.8 in this numerical example, will the rank reversal region for sophisticated agents separates from that of naive agents. The same is true for asset choices. As shown above, only when the self-control problem is severe (β is very small) whether the agent recognizes self-control problem or not makes a behavioral difference. As $\beta \to 1$, both models converge to the exponential model.

5 A Multi-period Model with Uncertainty

A dynamic model is presented in this section, which captures the consumer decision problem in the market experiment more realistically. Compared with the previous model, this dynamic model adds four realistic institutional features. First, consumers face uninsurable income risk, both transitory and persistent. Second, consumers receive new introductory offers from other credit card companies every period. Third, receiving new offers is a probabilistic event. Consumers have a rational assessment of what the probability of new offers. Fourth, consumers have a time varying switching cost every period. The last feature captures the variance of consumers' personal time schedule or the variance of their emotional status, which determine consumer perceived switching costs.

These realistic features are added to explore the possibility of explaining "rank reversal" by random shocks, not time inconsistent preferences. Consumer preference for the short offer A may be optimal based on their expectation about future, though not according to the true realization. For example some consumers may have chosen the short offer under the expectation that they will switch out after six months. However, they then fail to transfer because they are too busy.

The model is inspired by standard "buffer-stock" life-cycle models, Carroll (1992, 1997), and Deaton (1991). This model is set in discrete time. One period in the model represents one quarter in the real world. The consumer lives for T periods. The boundary condition is that the consumer consumes all her cash-on-hand in the final period.[12] The consumer receives stochastic income every period. She can either save in her saving account or borrow on credit cards to smooth her consumption. She is liquidity constrained in two respects. First, she is restricted in her ability to borrow. The upper bound is the total credit limit of her credit cards, denoted as $\bar{L}$, which is exogenously given. However nothing prevents her from accumulating liquid assets. Second, she faces different interest rates depending on whether she is saving (r^s) or borrowing (r), where $r > r^s$, and r is the regular interest rate on credit cards.

The consumer can reduce the interest payment on her debt if she accepts an introductory offer. At the beginning of period 1, the credit card company that has conducted this market experiment, denoted as Red, offers the consumer an introductory interest rate $r^R < r$ with a duration τ^R periods, and a credit limit l. The consumer may also receive credit card solicitations from other credit card companies that are not observed in this dataset. These unobservable companies are simplified as one company, Blue. Blue provides an introductory interest rate $r^B < r$ with an introductory duration of τ^B and a credit limit also l. Credit lines from both companies should be

[12]The model is chosen to have a finite horizon because the standard contraction mapping theorem fails for sophisticated hyperbolic models. See Laibson (1997,1998) for more details. We choose T large enough so that results will not be sensitive to the time horizon.

almost the same because credit lines are determined by consumer bureau information which is the same to both companies. The consumer's total credit limit $\bar{L}$ is held constant even after accepting a new offer to simplify computation. This should be an innocuous assumption since the whole sample is definitely not line constrained. Recall consumers in the sample have so much unused credit limit even at the time of solicitation: the average debt is only \$2,500 while the average revolving limit is \$15,000 on credit cards. In every period, the consumer receives a Blue offer with a probability q, which is positive and finite if the consumer has no existing introductory offer from Blue, otherwise zero. [13]

There is a switching cost k_t associated with accepting every introductory offer. The switching cost is indexed by t because it is assumed that the consumer has a time-varying switching cost. Normally switching costs are simply assumed to be constant, such as Sorensen (2001) and Kim, Kliger and Vale (2003). The assumption is reasonable in those research because randomness will only add analytical complexity without more benefit. However this assumption is not realistic, because the disutility from switching critically depends on consumer personal schedule and emotional condition at the time of receiving solicitation. Simulation results show that sophisticated treatment of switching costs is required to explain the second puzzle, respondents with similar credit card debt fail to switch after this Red offer expires. Both models, hyperbolic and exponential, predict the respondents should switch out after the offer expires if switching costs are constant. This is because that both models are stationary. If it is worthwhile to accept the first offer so it is to the second offer. With random switching cost, respondents of this experiment accept the Red offer due to their low realized switching costs at the time of solicitation. However, their mean switching costs are much higher, which can be partially inferred from the low response rate (1%). This high mean will keep the majority of respondents from switching a second time after the introductory periods. It is assumed that there is no extra cost for transferring balances after the consumer accepts a new offer. Once she accepts the introductory offer(s), she has immediate access to the credit. Simultaneously with the acceptance of her credit card offer(s), the consumer decides how much to consume at the beginning of period t.

The consumer in period t maximizes a weighted sum of utilities from the current period on,

[13] This assumption effectively assumes that consumers have no more than one introductory offer from Blue. We believe relaxing this assumption will only complicate the problem with little benefit.

which is summarized in the following Eq.(**??**).

$$
\begin{aligned}
V_{t,t}\left(\Lambda_t\right) &= \max_{C_t, d_t^B, d_t^R} \tfrac{C_t^{1-\rho}}{1-\rho} - d_t^B k_t - d_t^R k_t + \beta_0 \delta E\left\{\widetilde{V}_{t,t+1}\left(\Lambda_{t+1}\right)\right\}, \quad \text{for } t = 1, \\
V_{t,t}\left(\Lambda_t\right) &= \max_{C_t, d_t^B} \tfrac{C_t^{1-\rho}}{1-\rho} - d_t^B k_t + \beta_0 \delta E\left\{\widetilde{V}_{t,t+1}\left(\Lambda_{t+1}\right)\right\}, \qquad\qquad \text{for } t \geq 2.
\end{aligned}
\tag{3}
$$

The instantaneous utility is the sum of the consumption utility and the disutility (the switching cost) from accepting an introductory offer. C_t and d_t^B are the consumption choice and the decision to accept an introductory offer from Blue in period t respectively. d_1^R is the decision to accept the Red offer in period 1. The consumption function is assumed to be CRRA and ρ is the coefficient of relative risk aversion. Λ_{t+1} denotes the vector of state variables: $\left\{X_{t+1}, \varphi_{t+1}, k_{t+1}, \tau_{t+1}^B, \tau_{t+1}^R, s_{t+1}\right\}$. X_{t+1} is cash-on-hand at the beginning of period t+1, which is a sum of stochastic income, y_{t+1}, and wealth, A_{t+1}. φ_{t+1} is the realized persistent income shock at period t+1, which will be discussed in more detail later. k_{t+1} is the realized switching cost in period t+1. τ_{t+1}^B and τ_{t+1}^R denote the number of introductory periods left on the Blue and Red card in period $t+1$ respectively. s_{t+1} denotes whether a new introductory offer is received in period $t+1$. The expectation is taken with respect to the distributions of y_{t+1}, φ_{t+1}, k_{t+1} and s_{t+1}.

$\widetilde{V}_{t,t+1}$ is a weighted sum of self t's excepted future utilities and it is recursively defined as:

$$
\widetilde{V}_{t,t+1}\left(\Lambda_{t+1}\right) = \tfrac{\widehat{C}_{t+1}^{1-\rho}}{1-\rho} - \widehat{d}_{t+1}^B k_{t+1} + \delta E\left\{\widetilde{V}_{t+1,t+2}\right\} \tag{4}
$$

$\widehat{C}_{t+1}$ and $\widehat{d}_{t+1}^B$ are the behavior of the expected self $t+1$. Self t takes the expected self $t+1$'s behavior as given so that there is no 'max′ operator in Eq.(**??**). The discount factor between period t and $t+1$ is δ because the relative discount factor from self t's point of view is δ. $\widetilde{V}_{t+1,t+2}$ is used instead of $\widetilde{V}_{t,t+2}$ because self t and self $t+1$ have the same expectation about periods later than $t+1$. This special feature of quasi-hyperbolic models makes it easier to compute.

$\widehat{C}_{t+1}$ and $\widehat{d}_{t+1}^B$ are determined by the following optimization problem where the expected future discount function is used:

$$
\max_{\widehat{C}_{t+1}, \widehat{d}_{t+1}^B} \frac{\widehat{C}_{t+1}^{1-\rho}}{1-\rho} - \widehat{d}_{t+1}^B k_{t+1} + \beta_1 \delta E\left\{\widetilde{V}_{t+1,t+2}\left(\Lambda_{t+2}\right)\right\} \tag{5}
$$

This consumer problem is solved numerically by backward induction. One can iterate Eq.(**??**) and Eq.(**??**) to generate the expected continuation utility function $\widetilde{V}_{t,t+1}$, then combine the $\widetilde{V}_{t,t+1}$ and Eq.(**??**) to calculate decision rules for C_t, $d_1^R$and d_t^B.

6 Estimation

In this section, we will apply the dynamic model to the empirical data and estimate related parameters.

6.1 Estimation Strategy

The Maximum Likelihood function implied by the above dynamic model is very complicated due to two reasons. First, there is an endogenous sampling in the first period: we only observe respondents' subsequent borrowing behavior, who are 1% of the original population. To account for this endogenity, the likelihood function will involve high dimensional integration which is computational prohibitive. Second, there are two choice variables in the model, one of which is a continuous variable (consumption). The likelihood function for a continuous variable is also difficult to compute. To circumvent these problems, the parameters of the model are estimated by matching empirical moments with simulated moments from the dynamic model. The estimation method used is *Simulated Minimum Distance Estimator* (SMD), proposed in Hall and Rust (2002). [14]

The SMD estimator is the parameter value θ that minimizes the distance between a set of simulated and sample moments. The sample moments are calculated based on censored observations, the respondents. Consumers' behavior is simulated for a given trial value θ and simulated moments are also based on respondents, who are censored in exactly the same way as in the empirical data. Even though various moments based on censored data may be biased, the SMD estimator is consistent as proved in Hall and Rust (2002).

Denote the empirical moments we want to match as

$$h\left(\theta^{*}\right)=\sum_{i=1}^{N}\frac{1}{N}h_{i}\left(\theta^{*}\right)$$

which is the sample mean and θ^{*} is the underlying true parameter vector. The simulated moments are a function of the parameter vector θ, as

$$h\left(\theta\right)=\sum_{i=1}^{N}\frac{1}{N}h_{i}\left(\theta\right)$$

[14] This method is similiar to Simulated Moments Estimator (SME) of McFadden (1989) and Pakes and Pollard (1989).

The simulated minimum distance estimator $\widehat{\theta}$ is to minimize a weighted distance between the simulated moments and the sample moments, as defined by:

$$\widehat{\theta} = \arg\min_{\theta\in\Theta} \left(h\left(\theta\right) - h\left(\theta^*\right)\right)' W \left(h\left(\theta\right) - h\left(\theta^*\right)\right)$$

In this study, a total of 216 moments are used, 36 for each market cell. The 36 moments are the response rate plus 35 debt moments (five debt distribution statistics for seven quarters: the proportion of consumers who borrow, mean, median, fortieth and sixtieth percentiles among borrowers). The debt statistics for the first quarter are omitted because they underestimates respondents' debt. This underestimation occurs because it takes about 2-3 months for respondents to accumulate debt on this card. This time lag is not modeled in the dynamic model.

So many debt moments are chosen because simulation results indicate that they are critical for accurately identifying model parameters. The cost of so many debt moments is that the corresponding optimal weighting matrix[15] gives too much weight to the debt path. The optimal set of parameters minimizes the distance from debt at the cost of response rates. However as shownn above response rates are crucial in distinguishing between hyperbolic and exponential discounting. Therefore we decide to use a non-optimal diagonal matrix as the weighting matrix. Resulting estimates are still consistent but not efficient. Our weighting matrix puts 80% weight on response rates and 20% on the debt path moments. Among six market cells, 1/3 of the weight is on cell A, 1/3 is on F and the remaining 1/3 is equally distributed among B, C, D, E.

The asymptotic distribution of estimated $\widehat{\theta}$ is

$$\sqrt{N}\left(\widehat{\theta} - \theta^*\right) \rightarrow N\left(0, 2\Lambda_1^{-1}\Lambda_2\Lambda_1^{-1}\right) \tag{6}$$

where

$$\Lambda_1 = \nabla Eh\left(\theta^*\right)' \cdot W \cdot \nabla Eh\left(\theta^*\right),$$
$$\Lambda_2 = \nabla Eh\left(\theta^*\right)' W \cdot \Omega\left(h\left(\theta^*\right)\right) \cdot W \cdot \nabla Eh\left(\theta^*\right),$$

[15] Hall and Rust (2002) proves that the optimal weighting matrix W^* is

$$\begin{aligned} W^* &= \left(\Omega\left(h\left(\theta^*\right)\right)\right)^{-1}, \\ \text{where } \Omega\left(h\left(\theta^*\right)\right) &= \frac{1}{N}\sum_{i=1}^{N}\left(h_i\left(\theta^*\right) - h\left(\theta^*\right)\right)\left(h_i\left(\theta^*\right) - h\left(\theta^*\right)\right)' \end{aligned}$$

W is the weighting matrix, $\Omega\left(h\left(\theta^*\right)\right)$ is the variance matrix of the moments, and $\nabla Eh\left(\theta^*\right) = \frac{\partial Eh(\theta^*)}{\partial\theta^*}$. The over-identification X^2 statistics is:

$$\frac{T}{2}\left(h\left(\widehat{\theta}\right)-h\left(\theta^*\right)\right)' P^{-1}\left(h\left(\widehat{\theta}\right)-h\left(\theta^*\right)\right)$$

where

$$P=\left[I-\nabla Eh\left(\theta^*\right)\Lambda_1^{-1}\nabla Eh\left(\theta^*\right)'\cdot W\right]\cdot\Omega\left(h\left(\theta^*\right)\right)\cdot\left[I-\nabla Eh\left(\theta^*\right)\Lambda_1^{-1}\nabla Eh\left(\theta^*\right)'\cdot W\right].$$

6.2 Calibration of parameters

To make estimation feasible, we calibrate a subset of parameters, using related literature and our dataset, and make assumptions about exogenous variables' distributions. First is the income process, which is modeled as a time series with persistent and transitory shocks. The persistent income shock is captured by a two-state Markov process, $\varphi_t \in \{1,0\}$, where 1 and 0 represent the good and bad state respectively. This process was introduced in Laibson *et al.* (2000) and adopting it significantly reduces the computational cost. The transition probabilities between the two states are governed by the conditional probabilities matrix: $\{p_{i,j}\}$, where $i,j\in\{1,0\}$, $p_{i,j}=prob(\varphi_t=i/\varphi_{t-1}=j)$. In a given state, income is a random draw from a lognormal distribution, $LN\left(\eta^j,\varepsilon^j\right)$, where $j\in\{g,b\}$. The lognormal distribution captures the transitory income shock and its parameters depend on whether the persistent state is good or bad. To get reasonable estimates for the income distribution, we use estimates from Laibson et al. (2000) as a starting point. We describe the detailed calibration of income process in the Appendix.

We assume that the switching cost, k_t, is an iid random draw from a uniform distribution with range $[0,k]$. We assume consumer liquid asset/credit card debt at the time of solicitation is drawn from a normal distribution with a mean of μ and a variance of ϵ^2.

We calibrate the total credit limit, $\overline{L}$, and the credit limit for each card l, using the information in the dataset. The calibrated $\overline{L}$ are \$15,000 and l is \$6,000. In addition, the regular interest rate for credit cards r is assumed to be 1.16%, and the saving interest rate $r_s = 1.01\%$. The relative risk aversion coefficient ρ is assumed to be 2.

The introductory interest rates and durations of the Red offers, r_R and τ_R are given in the experiment dataset. However, we don't observe introductory offers consumers received in subsequent periods. We assume the duration for the Blue offer is 6 months, which is the typical duration for the company we observed. The interest rate on the Blue offer is assumed to be 8%.

We assume consumers have a probability of 90% of receiving Blue offers. As argued before, we believe respondents receive new offers every quarter with a probability of almost one in the sample period. It is assumed that 1% consumers have an ongoing Blue offer at the time of the Red solicitation, based on the average response rate to credit card solicitations during the sample period.

Given the calibrated parameters and the distribution assumptions, we estimate the remaining parameters by SMD. The estimated parameters are the time discount factors, β and δ, the switch cost distribution parameter k, and the parameters of the liquid asset distribution at the beginning of period 1, the mean μ and the variance ϵ^2. We estimate parameters for three models: exponential, naive and sophisticated hyperbolic. For the exponential model, $\beta = 1$. For the naive model, $\beta_1 = 1$ and $\beta_0 = \beta$ is estimated. For the sophisticated model, $\beta_1 = \beta_0 = \beta$. Standard errors are calculated according to Eq.(**??**).

6.3 Numerical Simulation and Model Prediction

Before presenting estimation results, we use some numerical simulations to provide intuition about the model behavior. All simulations are based on the calibrated parameters.

It is well-documented that sophisticated hyperbolic models have irregular policy functions as the short-term discount factor β becomes smaller. (Krusell and Smith 2000, Harris and Laibson 2001a). The irregularity is due to strategic interactions between selves at different periods. Due to time inconsistent preference, an early self desires different actions from a later self than what the later self will actually do. Therefore, the early self will behave strategically, trying to align the later self's behavior as closely as possible to what she wishes.

Fig.(**??**) plots the asset choice A_{t+1} as a function of cash-on-hand at period t, X_t for different β, given $\delta = 0.9999$ and k=0.03. The left graph is for the naive hyperbolic model and the right is for the sophisticated model. The asset function of the naive model is regular for all β values, since naive agents don't recognize time inconsistency. However when $\beta = 0.5$, the asset function of the sophisticated model is a step function, which is quite irregular. Converting asset functions into consumption functions, the regular asset function corresponds to a concave, monotonic consumption function. However the step function will generate a non-monotonic consumption function. When β is close to 1, the asset functions for the naive model are similar to those for the sophisticated model, consistent with our finding in the complete information model. When $\beta = 0.5$, the two models behave qualitatively different, although they have a similar mean asset choice. The sophisticated asset function varies around the naive one as shown in Fig.(**??**).

Can random shocks explain "Rank Reversal"? Simulation results reveal that the conflict between preference for the short offer A and later low switching is still inexplicable in the exponential model. In the top panel of Table (**??**), exponential agents' response to offer A and F are reported for different δ, given other parameters. The more patient the agents are, the more response to the short offer A compared with that to offer F. Patient consumers expect that their debt will be short-lived so that the shorter offer A is better. On the contrary, agents are more likely to accept the longer offer when they become impatient. At the same time, impatient respondents will be more likely to stay with the card after the interest rate jumps to 16%. The corresponding average debt for respondents over time are shown in Fig.(**??**). The time consistent agents always prefer an offer incurring the least cost. The short offer costs less only if the debt declines rapidly over time. Under that scenario, earlier interest saving can compensate for the later higher interest rate. Therefore there doesn't exit a δ, which can simultaneously explain the two phenomena.

Simulation results for the sophisticated and naive are reported in Table (**??**), Fig.(**??**) and Fig.(**??**), where $\delta = 0.9999$. When $\beta = 0.8$ both sophisticated and naive models predict that agents prefer the short offer and they keep on borrowing on the card for a long period. Only when $\beta = 0.7$, the naive model behaves significantly different from the sophisticated model. Naive consumers prefer the short offer A to the longer offer F because they naively believe that their debt is short-lived. sophisticated consumers, however, prefer the longer offer because it saves much more interest than the short one, which is outweighing the benefit of constraining future selves.

6.4 Estimation Results

Estimation results for the dynamic model are reported in Table (**??**). "Goodness-of-Fit" is the weighted distance between empirical moments and simulated moments. Allowing for hyperbolic time preferences significantly improves fit, reducing the distance by more than half. As explained above, the failure of exponential discounting is expected because the exponential model cannot simultaneously explain consumer response to different offers and respondents' later borrowing behavior. Even after random shocks are incorporated into the model, time consistent consumers on average exhibit consistent behavior. Only by allowing consumers to have time inconsistent preferences, can the model prediction match the empirical data.

An inspection of Table (**??**) shows that all parameters are estimated precisely. The parameters for both hyperbolic models are very close, while those of the exponential model are quite different. As shown above, the sophisticated model is similar as the naive model when β is close to 1. There is only a small quantitative difference: given β and δ, naive consumers borrow more and are more eager to accept new offers. Therefore, the naive model needs a larger β to match the consumer debt level and a larger switching cost to keep consumers from switching out. The β estimates match with Laibson, Repetto and Tobacman (2004), whose $\beta = 0.7$.

In Table (**??**) the switching cost parameter k is transformed into a dollar value. k measures a utility value in the dynamic model. To interprete it intuitively, an approximate dollar value is calculated, dividing k by the marginal utility at the average consumption level among the solicited population. For example, $k = 0.0292$ corresponds to a dollar value of \$292. In another word, the mean of switching cost is \$146 which belongs to a uniform distribution $[0, k]$. The distribution parameters, μ and ϵ^2, are also transformed to facilitate estimation. Both estimated values and their corresponding dollar values are reported in Table (**??**).

Exponential consumers are estimated to have a much larger switching cost k, a larger mean μ and a larger variance ϵ^2. Such parameters are required to better match the debt path over time. To match preference for the shorter offer, exponential consumers have to have a large δ, 0.9999, as shown in the above simulation. Such patient exponential consumers are more likely to borrow under 16% APR only when they have a higher switching cost. With a higher switching cost, μ and ϵ^2 have to change accordingly to match the magnitude of average debt and response rates.

Even though hyperbolic parameters are accurately identified, the model is decisively rejected as shown in the large over-identification test statistics X^2. However hyperbolic models are rejected to a lesser degree compared with the exponential model. The failure of this test is due to two possible reasons. First the model is based on a large-sample dataset. About 100,000 observations contribute to each moment. As the sample size becomes so large, any small misspecification will result in rejection of the model. Second, we try to match so many moments that any small error in one moment will be adde up to a large error which will lead to reject the model. Nevertheless, our model still captures many interesting aspects of consumer behavior.

Consumer responses to six different introductory offers are shown in Table (**??**). All three models match the response rates due to a large weight on this moment. Hyperbolic models fit better than the exponential model because they also match the relative preferences among the six offers.

In Fig.(**??**), the predicted debt paths of Market Cells A, E and F implied by the three models are compared with empirical data. Comparing to the exponential model, the two hyperbolic models match the debt path much better, which is the reason why their "Goodness-of-Fit" statistics are much lower. Despite a very large switching cost, the debt path predicted by the exponential model declines much faster than the data. Exponential consumers borrow too much at the beginning, an average of \$3500 compared with \$2700 empirically, and too little at the end, an average of \$900 instead of \$2600 empirically. Such a debt path is predicted because that exponential consumers are so patient ($\delta = 0.9999$) that they will pay off their debt even without switching. However a large δ is required to match consumer preference for the short offer.

The magnitude of k deserves some discussion. Is the average switching cost \$150 outrageously high? The large magnitude of k is consistent with anecdotal evidence in the credit card market. Credit card issuers spend lots of money to acquire new accounts. Credit card companies send out billions of solicitations every year and 99% of them end up in trash cans. Many solicitations offer a very low introductory rate, as low as 0%. The behavior of issuers will only be rational if majority consumers don't switch. In contrast to low acceptance rates, the average credit card debt is \$9000 among U.S. households with at least one credit card.[16] This expensive inertia directly implies high disutility that consumers associate with card application, which is captured by switching cost (k) in the dynamic model.

6.5 Robustness

In this subsection, we check the robustness of the above findings. First, can a time consistent model match empirical moments as well as hyperbolic models if it has an extra parameter β? To answer this question, we estimate a model whose discounting function is $\left\{1, \beta\delta, \beta\delta^2, ...\right\}$ at period 1 (solicitation time) and $\left\{1, \delta, \delta^2, ...\right\}$ at all later periods. This model is time consistent because the

[16] Based on data from cardweb.com.

desired discount rate is the same (δ) between any two consecutive periods. But it has a one-time present bias factor at the card acceptance stage which may generate preference for the low-rate short offer A. The estimation result for this model is reported in the first column of Table (**??**). It is apparent that the extra parameter fails to improve fitness significantly. Actually, the parameter β only changes the model prediction to a small degree, judged from the large standard error of this parameter. The small impact is reasonable considering that β only affects behavior at period 1, not later six periods.

Secondly, do the above findings still hold if Market Cell A is ignored? The above estimation critically depends on the assumption that consumers behave the same no matter which market cell they belong to. One possible criticism is that consumers in Market Cell A may not be comparable to consumers in other cells because offer A (4.9% for 6 months) is an exceptionally good offer. The ordinary offer from the issuer is B (5.9% for 6 months). To address this concern we estimate the dynamic model based on consumer behavior on other five market cells. The estimation results are shown in the last three columns of Table (**??**). Still hyperbolic models match empirical moments better than the exponential model. When cell A is omitted, time inconsistency has reduced, therefore the difference between exponential and hyperbolic models has decreased. Specifically, the Goodness-of-Fit of the exponential model has improved because a lower δ is required to match response rates. And a lower δ improves prediction on debt path. The significant change for hyperbolic models are that a larger β is needed to match response rates because the time inconsistency is reduced.

7 Conclusion

This paper applies three different models of intertemporal choices, exponential, naive hyperbolic and sophisticated hyperbolic, to explain consumer behavior in a credit card market experiment. From this interesting experiment, two contradictory phenomena are observed. First, at the time of solicitation, consumers prefer an offer with a lower introductory interest rate (4.9%) and a shorter duration (6 months), to an offer with a higher introductory interest rate (7.9%) but a longer duration (12 months). The preference is puzzling since respondents pay a lower interest rate, *ex post*, under the longer introductory offer. We call this phenomenon “rank reversal”. Second, the majority of respondents do not switch out after the expiration of their introductory offers, even though their debt remains at the level as when they accept the offer. This is puzzling because there are many other offers available and the benefit of switching is as large as before.

We first use a multi-period complete information model to analytically prove that standard exponential consumers will not exhibit "rank reversal". Exponential consumers always prefer the credit card offer which incurs the least interest payment. However, if consumers are assumed to have time inconsistent preferences, such as the newly developed hyperbolic discounting, "rank reversal" is not a puzzle any more. We have explored two extreme types of hyperbolic discounting: naive and sophisticated. Both models can explain the data, however the underlying stories are different. Naive consumers mistakenly prefer the shorter offer because they underestimate their future borrowing. Sophisticated consumers prefer the shorter offer because it offers a self-commitment device. Previous laboratory experiments try to solicit consumer time preferences by offering them financial rewards at different times. Besharov and Coffey (2003), however, show that this can't identify time inconsistent preferences because both consistent and inconsistent consumers would behave the same — maximize their wealth. The market experiment studied here provides a unique angle to identify hyperbolic discounting.

Can an exponential model with realistic random shocks explain the above two puzzles? To address this question, a dynamic model with uncertainty is developed, in which consumers have either time consistent (exponential) or time inconsistent (hyperbolic) preferences and they are subject to realistic random shocks, such as income shocks. The estimation results based on this dynamic model show that only the hyperbolic model can explain the two phenomena simultaneously. The exponential model fails because that time consistent consumers would always prefer an offer which on average provides the lowest interest payment. Moreover, consumers have high switching costs with an average of \$150.

Consumer time consistency is an important question since different models have vastly different normative implications. For example a consumer piles up debt on her credit cards. She may do so because the pleasure of consumption today outweighs the interest payment tomorrow. Or she may do so because she has an impulse to overspend which is not valued from the long-run perspective, like the sophisticated agent. The two stories have different public policy implications. The first consumer just borrows the right amount. However, the second consumer would like somebody to bind her hands. It is crucial to distinguish between the two hypotheses.

Consumer behavior identified here also facilitates the understanding of competition anomalies in the credit card market. Instead of lowering interest rates, credit card issuers fiercely compete with each other by sending out "junk mail". 99% of direct solicitation mails end up in trash cans. Credit card companies offer ridiculously low introductory interest rates to acquire new customers, like 0% for 12 months. Nevertheless the post interest rate sticks around the prime rate plus 9.99%. All these strategies are optimal only if consumers don't switch. This study not only provides individual-level evidence of this inertia, but also identify two separate forces behind it: self-control problems and high switching costs. This inertia generates enormous consumer welfare loss. Consumers of U.S. have an average of $7,200 credit card debt among households with at least one credit card (87 million). Now the average interest rate is about 14%. The average interest rate will be lower if there are more rate surfers. Suppose the interest rate is 1% lower this translates into 6.26 billion annual interest saving for consumers.

Another interesting finding is that when the present bias factor β is close to 1, sophisticated and naive hyperbolic models behave similarly. We analytically prove this in a three-period model with certainty and confirm it numerically in the dynamic model. This finding is interesting because the two models describe two fundamentally different consumers. Naive consumers fail to recognize they have a time inconsistency problem. However, sophisticated consumers foresee their self-control problems. Naive models are as easy to compute as exponential models. On the other hand, sophisticated models may have multiple equilibria as proved in Krusell (2003). One direct application of this finding is that future research can use naive models to approximate sophisticated models if no welfare analysis is involved, because the empirically relevant β is close to 1.

A Calibration of Income Process

Laibson *et al.* (2000) models the idiosyncratic income shock, ξ_t, as a sum of a persistent shock, μ_t, and a transitory shock, ν_t. The persistent shock follows an AR(1) process with a coefficient α.

$$\xi_t = \mu_t + \nu_t,$$
$$\mu_t = \alpha\mu_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ and $v_t \sim N\left(0, \sigma_v^2\right)$. He estimated α, σ_ε^2, σ_v^2 for three different education levels. The parameters for "completed college" are used in the estimation.

Define a quarterly income shock, η_q, such that $\xi_t = \sum_{q=4(t-1)+1}^{4t} \eta_q$.

$$\eta_q = s_q + \epsilon_q,$$
$$s_q = f s_{q-1} + \gamma_q,$$

where s_q is a quarterly persistent shock with a coefficient of f. $\gamma_q \sim N\left(0, \sigma_r^2\right)$ and $\epsilon_q \sim N\left(0, \sigma_\epsilon^2\right)$.

It can be shown that:

$$\begin{aligned}
&4\sigma_\epsilon^2 = \sigma_v^2 \\
&\frac{1}{1-\alpha^2}\sigma_\varepsilon^2 = (4 + 6f + 4f^2 + 2f^3)\frac{\sigma_r^2}{1-f^2} \\
&\frac{\alpha}{1-\alpha^2}\sigma_\varepsilon^2 = (f + 2f^2 + 2f^3 + 4f^4 + 3f^5 + 2f^6 + f^7)\frac{\sigma_r^2}{1-f^2}
\end{aligned}$$

After obtaining parameters for the quarterly shock, I use a two-state Markov process to replace the s_q which follows an AR(1), following Laibson *et al.* (2000). The Markov process is symmetric taking two values $\{\theta, -\theta\}$, where $\theta = \sqrt{\frac{\sigma_r^2}{1-f^2}}$ and the transition probability $p = \frac{1+f}{2}$. In this way the Markov process matches the variance covariance of s_q.

Recall the income process in the dynamic model, $y_t = \varphi_t y_t^g + (1-\varphi_t)\, y_t^b$. y_t^j is lognormal random variable, where $j \in \{g, b\}$ and φ_t is a signal whether the income state is good or bad.

$$\log\left(y_t^g\right) = c + \theta + \epsilon_t$$
$$\log\left(y_t^b\right) = c - \theta + \epsilon_t$$

where c is a constant to capture the permanent income. To determine c, I assume the mean income is \$10,000 per quarter.

In summary, the income process in the good state has a mean of 10,000 and a variance of 3.5×10^5. The income process in the bad state has a mean of 7645 with a variance of 2.05×10^5. The transition probability matrix is:

$$p = \begin{pmatrix} 0.9939 & 0.0061 \\ 0.0061 & 0.9939 \end{pmatrix}.$$

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Table 1: Sample Statistics[a]

	N	Months On File	Number of Past-due	Revolving Balance	Revolving Limit	Number of Credit Cards	Income
All[b]	599,257	174	0.0197	$2,509	$17,481	3.77	NA
		(71)	(0.139)	(4058)	(11388)	(1.88)	
A	1073	126	0.0308	$3,927	$15,473	3.94	$44,180
		(76)	(0.1727)	(4979)	(10573)	(2.057)	(24051)
B	903	128	0.0266	$3,474	$15,137	3.81	$43,170
		(79)	(0.1609)	(4725)	(11112)	(2.101)	(25175)
C	687	114	0.0247	$3,543	$14,230	3.598	$42,253
		(77)	(0.1555)	(4901)	(11268)	(2.068)	(24437)
D	645	112	0.0248	$3,584	$14,075	3.557	$41,215
		(76)	(0.1557)	(4988)	(11703)	(2.07)	(25274)
E	992	125	0.0363	$3,694	$15,176	3.729	$43,830
		(76)	(0.1871)	(5066)	(11313)	(2.076)	(28733)
F	944	123	0.0222	$4,042	$15,107	3.807	$43,697
		(77)	(0.1476)	(5469)	(10688)	(1.98)	(26725)

[a]Standard deviations are in parentheses.

[b]Sample statistics are reported for all six market cells to save space. Due to randomization, the statistics are similiar across different market cells.

Table 2: Rank Reversal[a]

Market Cell	Number of Observations	Effective Response Rate	Rank by Response Rate	Effective Interest Rate	Rank by Interest Rate
A: 4.9% 6 months	99,886	1.073% (0.00033)	1	10.23%	3
B: 5.9% 6 months	99,872	0.903% (0.00030)	4	11.35%	4
C: 6.9% 6 months	99,869	0.687% (0.00026)	5	11.86%	5
D: 7.9% 6 months	99,880	0.645% (0.00025)	6	12.35%	6
E: 6.9% 9 months	99,890	0.992% (0.00031)	2	9.23%	2
F: 7.9% [b] 12 months	99,860	0.944% (0.00031)	3	8.32%	1
T-TEST		P-VALUES			
A vs. E		7.23%			
A vs. F		0.29%			

[a]Standard errors in parentheses.

[b]It should be briefly be explained why the calculated effective interest rate for market cell F (8.32%) slightly exceeded the stated APR of 7.9%. First, the author's calculations incorporated the first 13 months of the potential life of the account, in order to deal with some timing problems in the data. Second, the APR is twelve times the monthly interest rate and, so, omits monthly compounding. Third, the introductory interest rate is conditional on the card holder remaining current on his account; each market cell includes customers who went delinquent and lost the introductory rate.

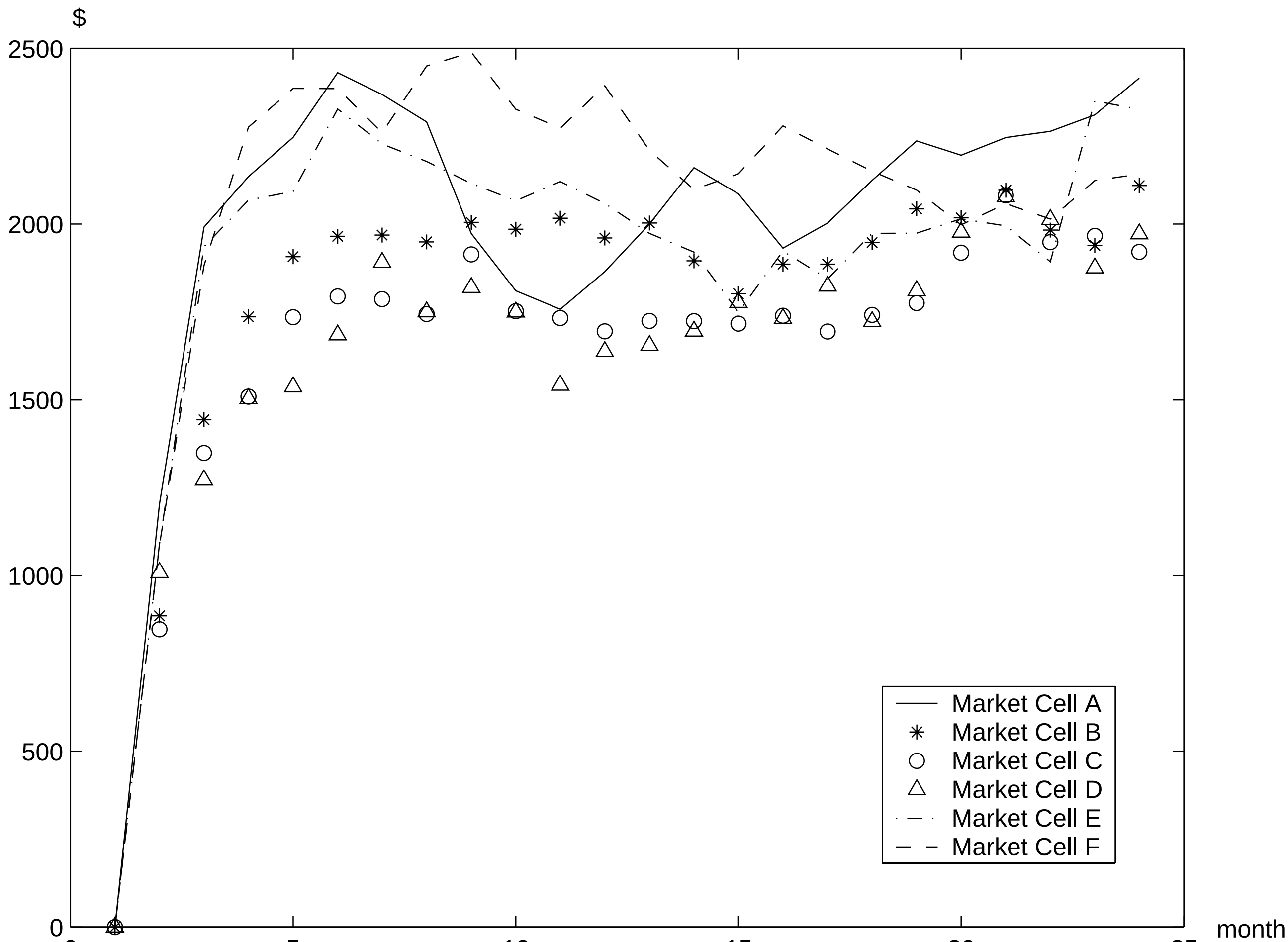

Figure 1: Medians of borrowers' debt distributions over time.

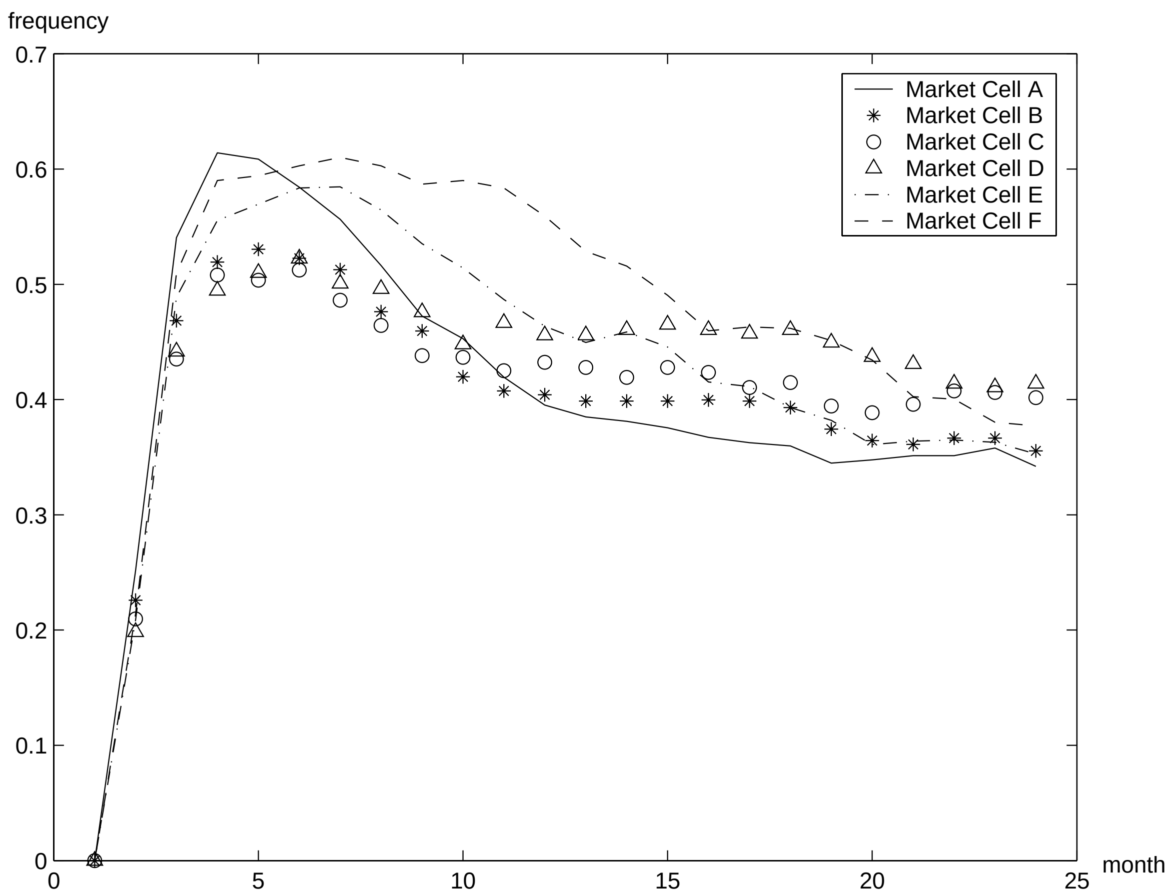

Figure 2: Borrowing Frequencies of respondents over time.

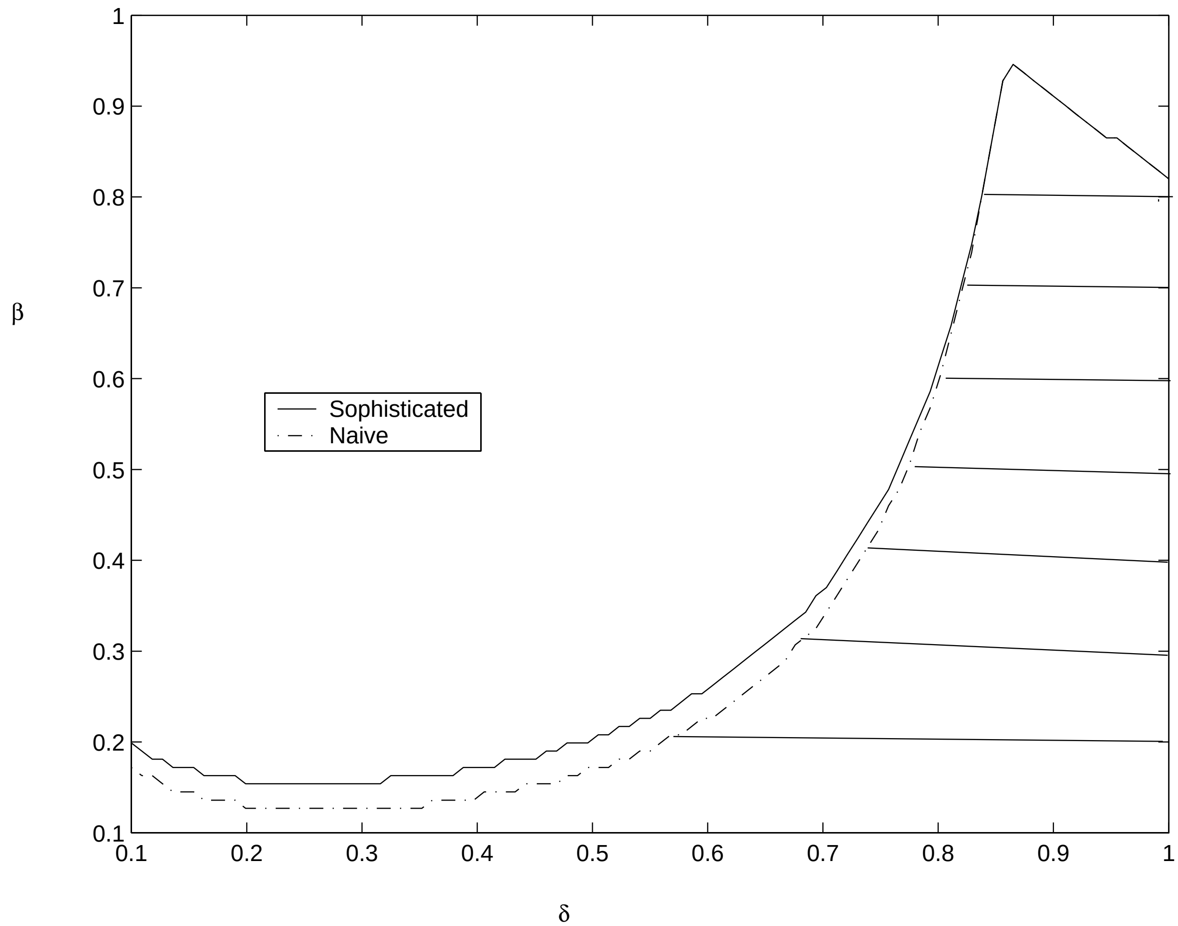

Figure 3: Rank Reverse Area of Sophisticated and Naive Hyperbolic Agents

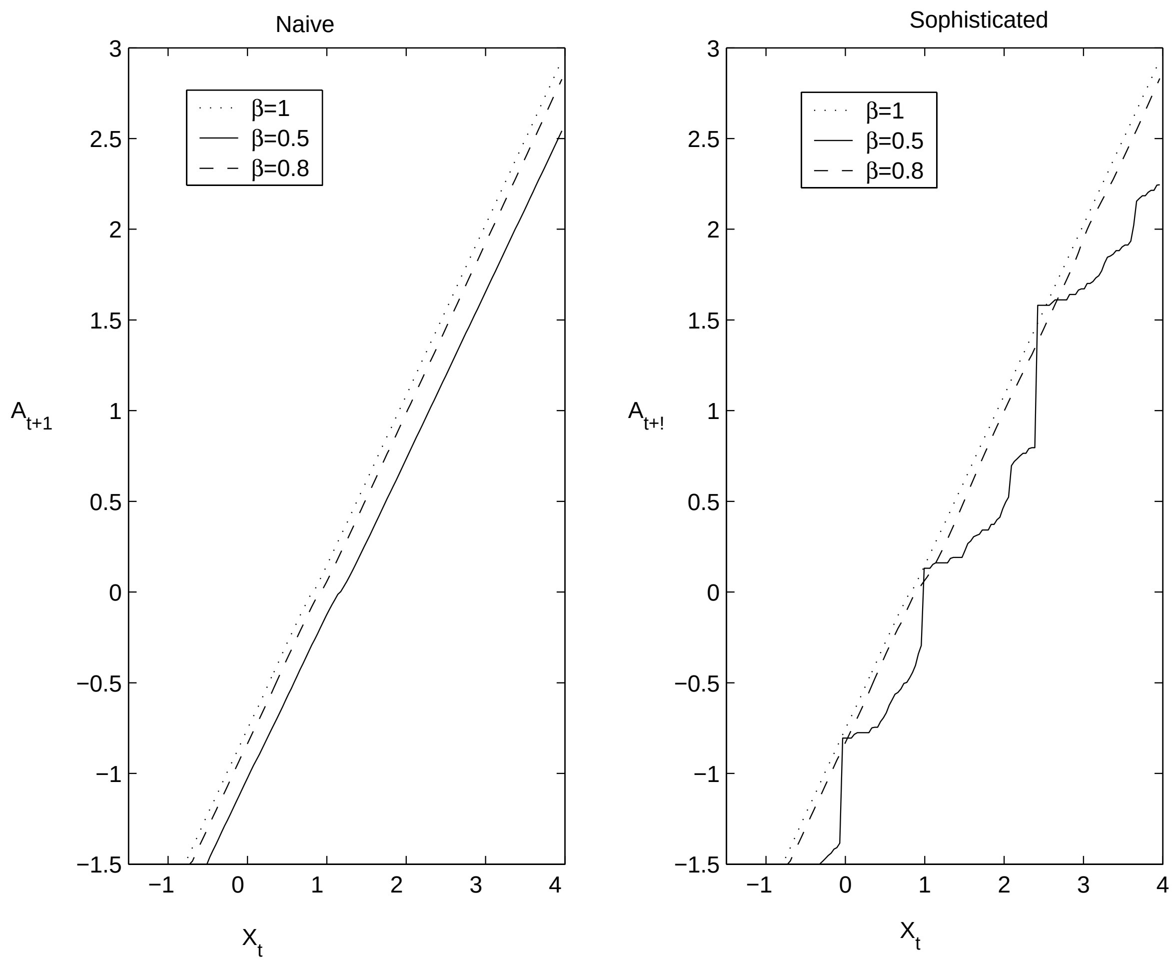

Figure 4: Policy Functions

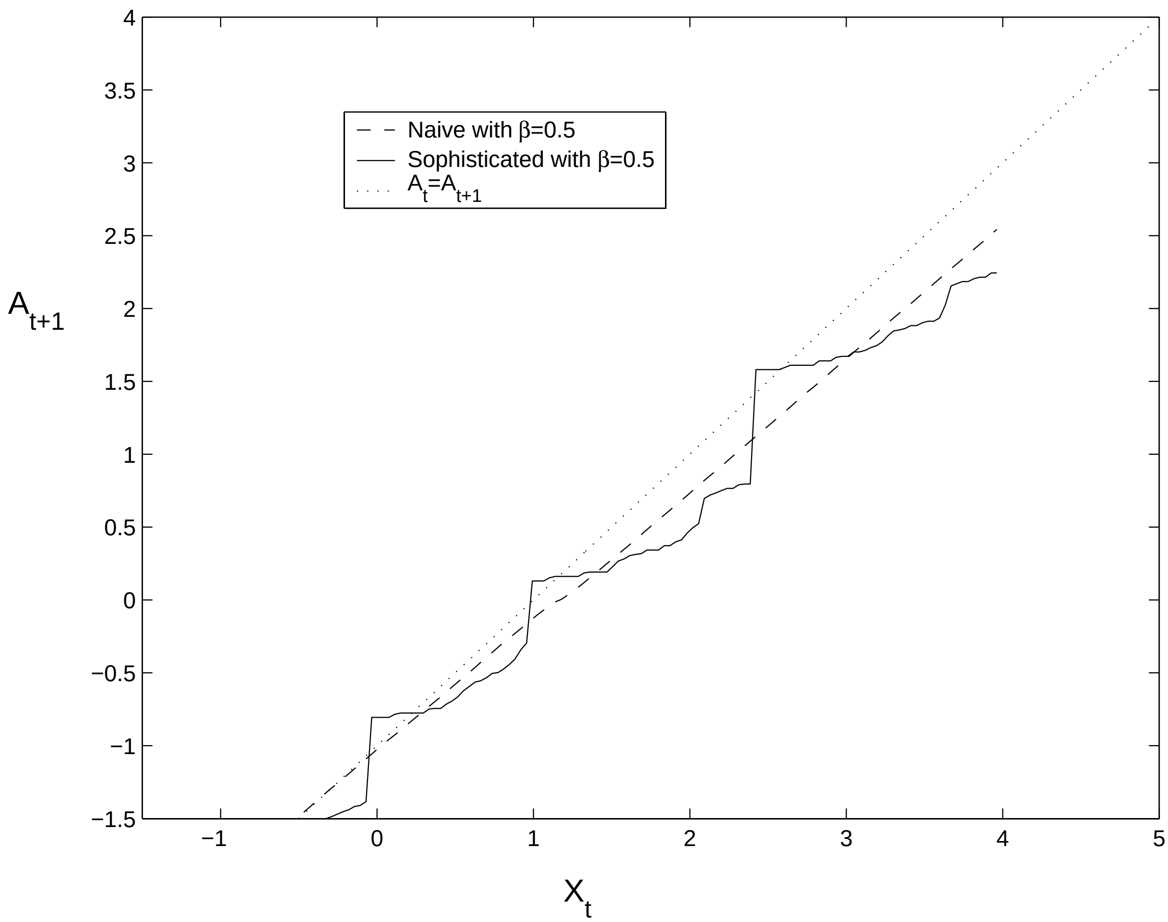

Figure 5: Policy Functions

Table 3: Rank Reversal Response [a]

Exponential				
δ	0.96	0.98	0.99	0.9999
A	152	72	53	41
F	476	100	71	36
Sophisticated				
β	0.7	0.8	0.9	0.95
A	98	80	54	47
F	133	72	41	49
Naive				
β	0.7	0.8	0.9	0.95
A	133	90	59	47
F	127	91	57	45

[a]Response is out of 10,000 simulations. β,δ are discount factors. k, the switching cost parameter, is 0.03. Mean of A_1 is 5000 and variance is 8e6, which is liquid assets at the time of solicitation.

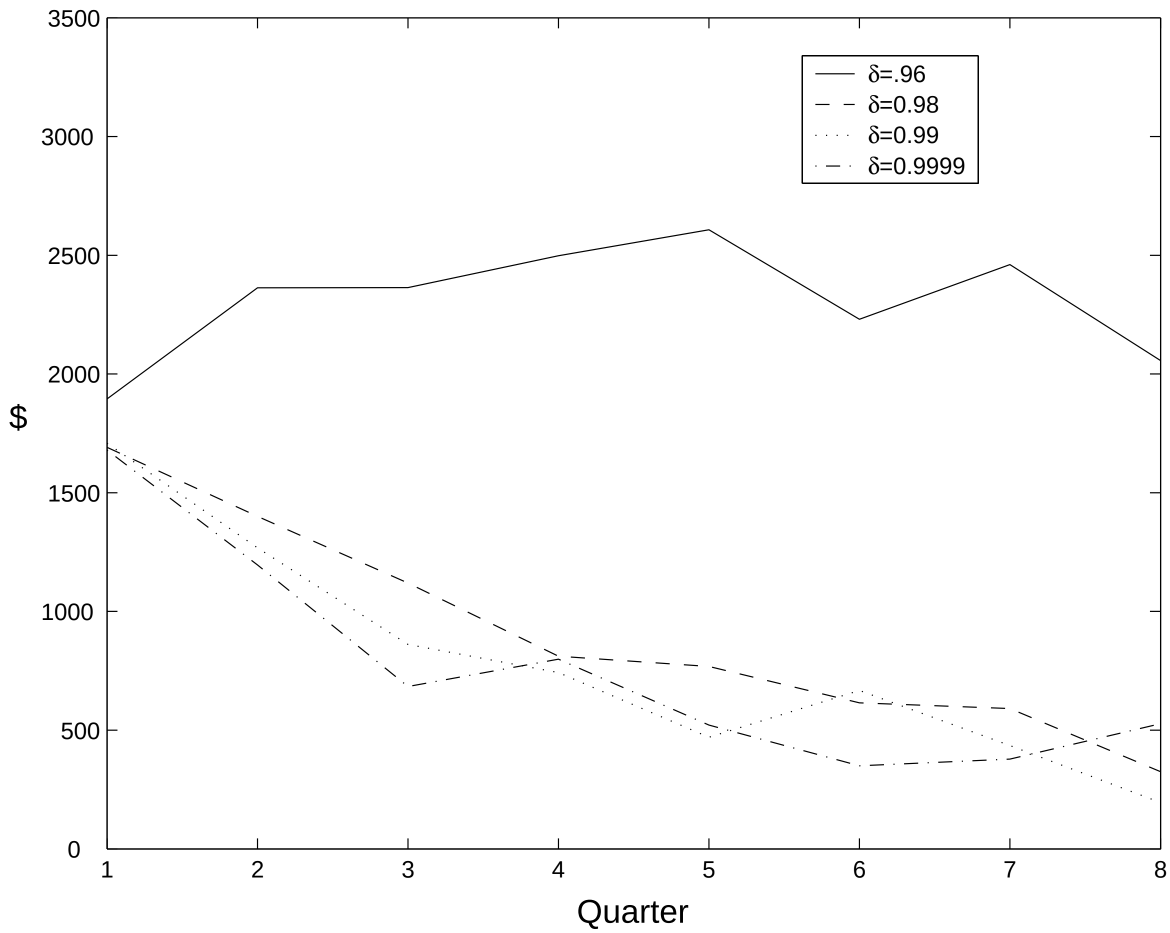

Figure 6: Simulated Mean Debt among responders of Market Cell A (Exponential)

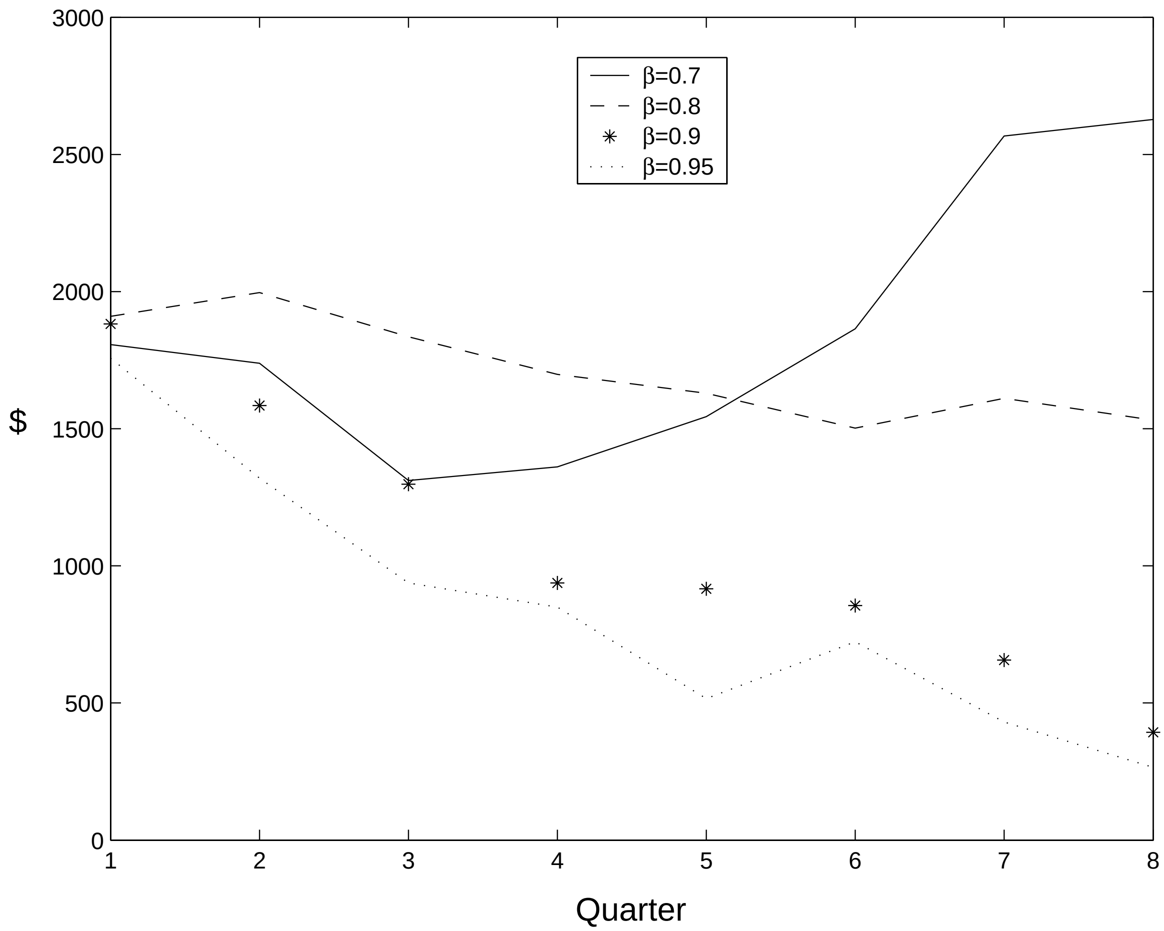

Figure 7: Simulated Mean Debt among responders of Market Cell A (Sophisticated Hyperbolic)

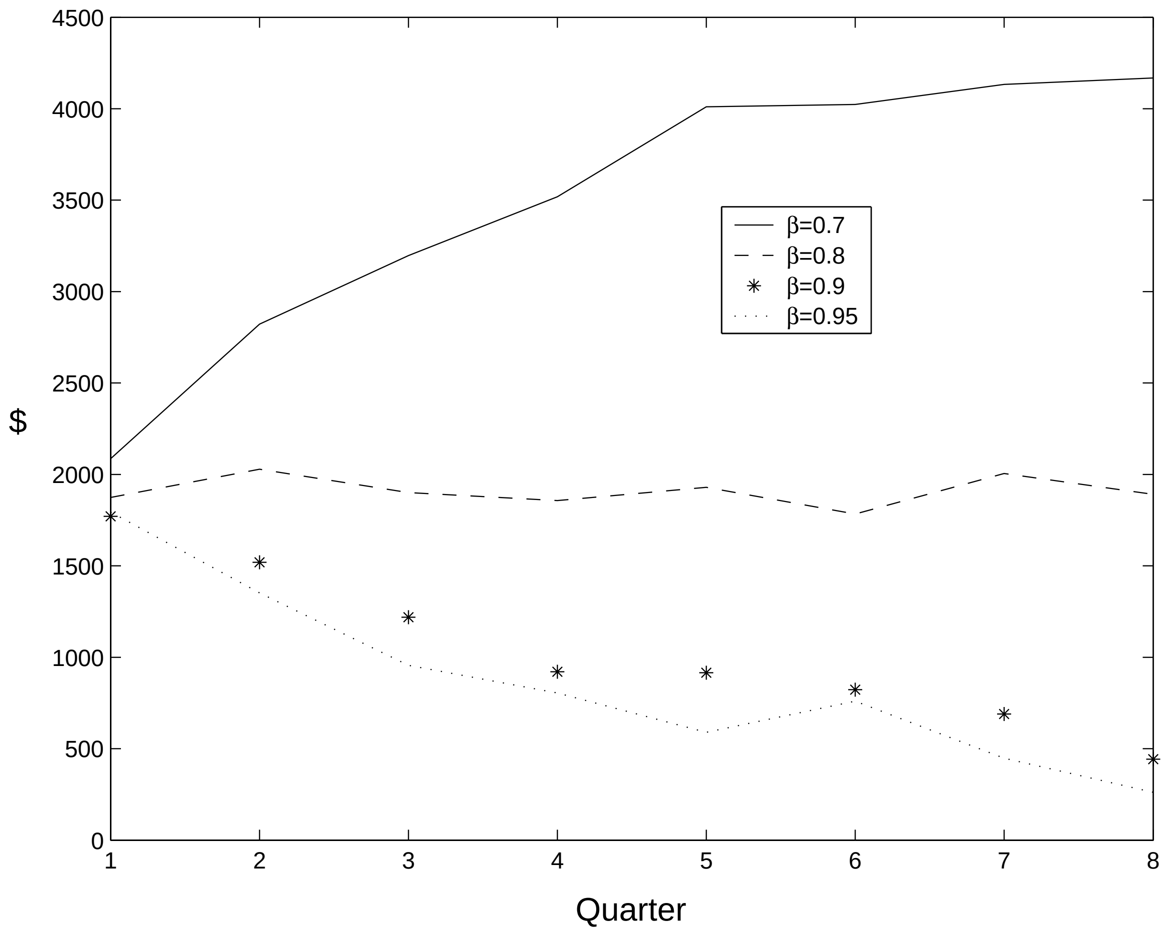

Figure 8: Simulated Mean Debt among respondents of Market Cell A (Naive Hyperbolic)

Table 4: Estimated Parameters [a]

	Sophisticated Hyperbolic	Naive Hyperbolic	Exponential
β	0.7863 (0.00192)	0.8172 (0.003)	
δ	0.9999 (0.00201)	0.9999 (0.0017)	0.9999 (0.00272)
k	0.02927 \$293 (0.00127)	0.0326 \$326 (0.00139)	0.1722 \$1,722 (0.0155)
μ	1.0088 \$5,044 (0.0367)	0.9584 \$4,792 (0.0277)	1.5836 \$7,918 (0.0517)
ϵ^2	0.831 \$8,310,000 (0.0439)	0.8167 \$8,167,000 (0.0373)	4.278 \$42,780,000 (0.0535)
Goodness-of-Fit	$2.5202e-4$	$2.8183e-4$	$6.0534e-4$
X^2	6110	5627	13975

[a] β,δ are discount factors. k is the switching cost parameter. A_1 is liquid assets at the time of solicitation. Standard errors are in parentheses.

Table 5: Simulated Response

Market Cell	Total	Empirical	Sophisticated Hyperbolic	Naive Hyperbolic	Exponential
A: 4.9% 6 months	99,886	1073	1001	1013	951
B: 5.9% 6 months	99,872	903	888	911	845
C: 6.9% 6 months	99,869	687	793	810	764
D: 7.9% 6 months	99,880	645	652	701	672
E: 6.9% 9 months	99,890	992	980	997	1005
F: 7.9% 12 months	99,860	944	947	978	1047

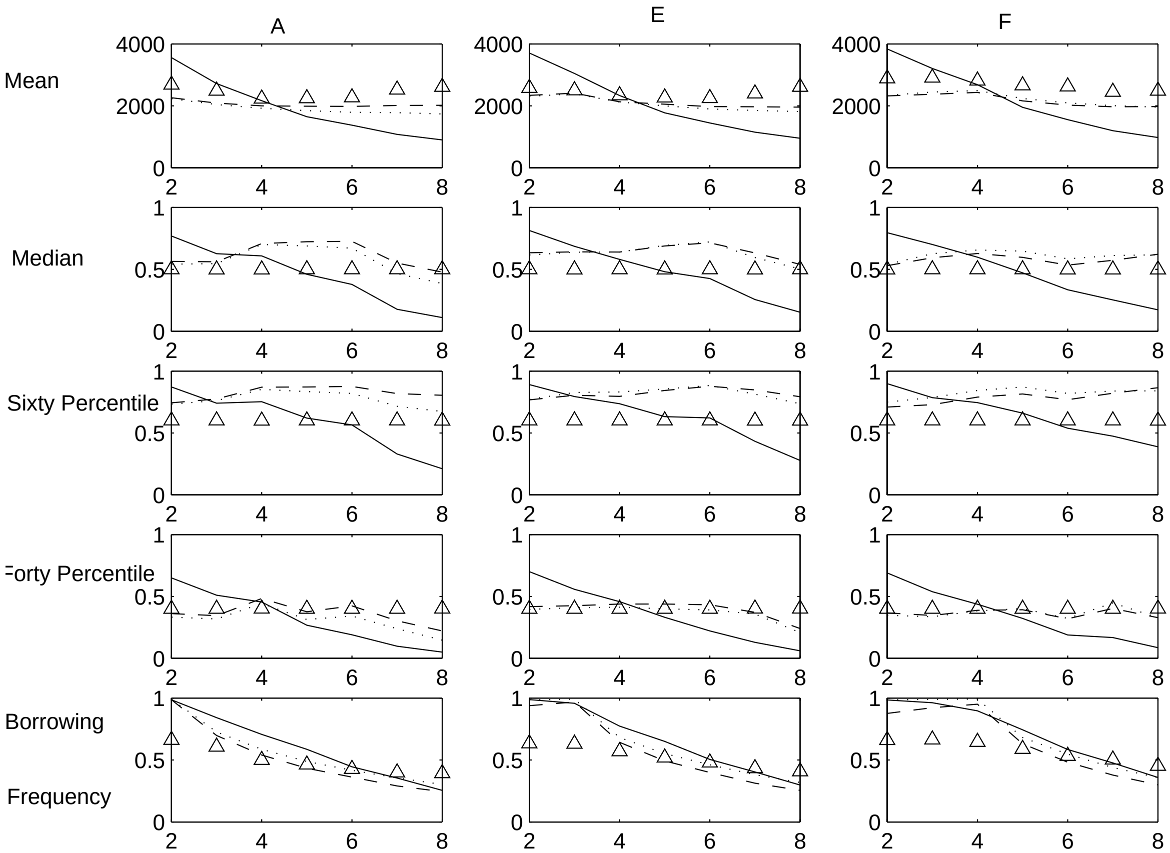

Figure 9: Simulated Debt Moments. The triangle line is the empirical data. The solid line, the dash line and the dotted line are predicted by the exponential, sophisticated and naive hyperbolic models respectively.

Table 6: Robustness [a]

	Consistent β	Sophisticated Hyperbolic	Naive Hyperbolic	Exponential
β	0.9999 (0.002967)	0.8034 (0.00253)	0.8246 (0.00255)	
δ	0.9999 (0.002972)	0.9998 (0.00133)	0.9999 (0.00124)	0.9726 (0.000962)
k	0.1722 $1,722 (0.0115)	0.0292 $292 (0.000945)	0.03266 $326 (0.00129)	0.0825 $825 (0.00358)
μ	1.5836 $7,918 (0.0422)	1.0072 $5,036 (0.0305)	0.9752 $4,876 (0.0346)	1.4534 $7,267 (0.02997)
ϵ^2	4.278 $42,780,000 (0.0437)	0.8377 $8,377,000 (0.0393)	0.8296 $8,296,000 (0.0514)	2.1854 $21,854,000 (0.055)
Goodness-of-Fit	$6.04892e-4$	$2.50492e-4$	$2.63716e-4$	$4.90161e-4$
X^2	20840	3996	4482	10610

[a] Market Cell A is excluded for Sophisticated Hyperbolic, Naive Hyperbolic and Exponential models. β,δ are discount factors. k is the switching cost parameter. A_1 is liquid assets at the time of solicitation. Standard errors are in parentheses.