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Market Design and the Evolution of the Combinatorial Clock Auction†

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The combinatorial clock auction (CCA) is an important recent innovation in market design. It has progressed rapidly from a 2003 academic paper to real-world adoption. In the past few years, it has been used for more major spectrum auctions worldwide than any other auction format. As such, the CCA is the first format that has the potential to eclipse the simultaneous multiple-round auction (SMRA) as the standard for spectrum auctions.[1]

The defining characteristic of the CCA is a two-stage bidding process. The first stage is a dynamic clock auction: the auctioneer announces prices for the items in the auction; and bidders respond with quantities desired at the announced prices. Bidding in this stage progresses in multiple rounds as prices increase until aggregate demand is less than or equal to supply for every item. In the second stage, bidders have the opportunity to submit a multiplicity of supplementary bids, both to improve upon their bids from the clock rounds and to express values for other packages.

Following the second stage, the bids from both the clock rounds and the supplementary round are entered into winner determination and pricing problems. The winner determination problem treats these bids as package bids, and determines the value-maximizing allocation of the items among the bidders. The pricing problem is based on second-price principles.

In most applications, there is also a third stage of bidding. Generally, several items in the auction are treated as identical during the first two stages. For example, in the European digital dividend auctions, there have generally been six distinct licenses offered in the 800 MHz band. In the auction's first two stages, bidders simply indicate quantities of "generic" 800 MHz spectrum that they wish to purchase. The third stage takes bidders' winnings of generic spectrum as given, and bidders express values for specific 800 MHz licenses. Thus, it determines the assignment from generic spectrum to physical frequencies.

The CCA addresses many concerns that had been raised in prior spectrum auctions. One clear weakness of the SMRA and other older auction formats has been that only single-item bids were permitted. A bidder who would achieve synergies from acquiring a New York license and a Washington, DC license is exposed to significant risk from bidding above the stand-alone value of either license; the bidder could be outbid on one license, while remaining obliged to purchase the other license. This is often referred to as the "exposure problem." The CCA eliminates the exposure problem by explicitly incorporating package bids. Another weakness of many older auction formats has been that the pricing rule creates incentives for strategic demand reduction and, consequently, has tended to produce inefficient outcomes. By moving in the direction of Vickrey pricing, the CCA mitigates incentives for demand reduction.

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[1] The CCA format was proposed by Ausubel, Cramton, and Milgrom (2006), first presented at the FCC's Wye River Conference in October 2003. The first practical implementations were the Trinidad and Tobago Spectrum Auction, in 2005, and the UK's 10–40 GHz and L-Band Auctions, in 2008. In recent years, it has been utilized for digital dividend auctions in Austria, Australia, Canada, Denmark, Ireland, the Netherlands, Slovakia, Switzerland, and the United Kingdom. Another account of the CCA is provided by Cramton (2013).

At the same time, the CCA's performance may be impeded by several limitations. First, with a large number of product categories (e.g., in spectrum auctions in countries with regional licenses), it may be difficult or impossible for bidders to express their values for all relevant combinations of items in the supplementary round. Second, the activity rules imposed on bidders may at once be too stringent, preventing straightforward bidding, and too weak, permitting manipulative and exploitative pricing of opponents. Third, unlike most other auction formats, the current CCA is "iterative second-price" rather than "iterative first-price": bidders generally pay less than the amounts of their winning bids. This creates a fundamental tension between the clock rounds and the supplementary round.

Since its initial proposal in 2003, the CCA has been in almost continual evolution. In this paper, we review a few of the most important changes that have already occurred and we propose three additional enhancements.

I. Early Evolution of the CCA

This section reviews a few of the most important evolutionary changes to the CCA.

A. *Opportunity Cost Pricing*

Since the CCA's inception, opportunity-cost pricing has been one of its main principles. Opportunity cost is formalized by the Vickrey-Clark-Groves (VCG) mechanism. However, to avoid "uncompetitive" pricing, the initial CCA proposal, as well as all implementations to date, have employed variants on the VCG mechanism that generate core payoffs (relative to the submitted bids).[2]

Several recent academic papers have emphasized the desirable incentive properties of the bidder-optimal frontier of the core (Day and Raghavan 2007; and Day and Milgrom 2008). Mechanisms selecting outcomes from this frontier are referred to as "core-selecting mechanisms." Due to the multiplicity of bidder-optimal core outcomes, Day and Cramton (2012) suggested selecting the bidder-optimal core point that minimizes the Euclidean distance from VCG payoffs. This is known as the "nearest-Vickrey" mechanism.

This choice of pricing rule has not been entirely uncontroversial. Several papers have contrasted the performance of nearest-Vickrey with other core-selecting mechanisms in a stylized sealed-bid environment and have argued that other mechanisms perform better.[3] However, these results are not necessarily applicable to the CCA, an auction format that includes a dynamic clock stage.

In practice, nearest-Vickrey pricing or a weighted version of this mechanism has been used in all CCA implementations to date.[4]

B. *Reserve Prices*

Reserve prices have been employed in most spectrum auctions, irrespective of format. The initial CCA proposal was not explicit about the reserve price implementation. Day and Cramton (2012) observed that a relevant design choice is whether the reserve prices are applied at the item level or at the package level. If applied at the item level, it is *as if* the auctioneer includes a collection of single-item bids, each bid from a distinct fictitious bidder, at the reserve price ("reserve bidders"). If applied at the package level, the auctioneer merely requires that the payment for each winning package bid must be at least the price of the items in the package evaluated at the reserve prices ("bounds only").

Day and Cramton (2012) favored the "bounds only" over the "reserve bidders" approach, for at least two reasons. First, holding reserve prices and actual bids fixed, the reserve bidders approach is more likely to result in items being withheld by the seller. Second, outcomes determined by the bounds only approach are less sensitive to choices of reserve prices. Largely on this basis, early CCAs in the United Kingdom and elsewhere adopted a bounds only approach.

However, under the bounds only approach, bidders might be able to acquire marginal items at very low incremental costs (sometimes zero).

[2] When goods are not substitutes, VCG prices may lie outside the core. To avoid this, the initial CCA proposal used the "ascending proxy auction" (Ausubel and Milgrom 2002; and Parkes 2001).

[3] See Erdil and Klemperer (2010) and Ausubel and Baranov (2013).

[4] Roughly speaking, the weighted version allocates the core burden among winners based upon the relative size of their winnings.

This violates one of the general rationales for a reserve price in a public auction: the reserve price should reflect the societal opportunity cost of selling the item today rather than saving it for later. Moreover, bidders in some scenarios may find it optimal to bid above their values for some items, knowing that they have unspent "reserve capacity" which will absorb the cost. This is not possible under a reserve bidders approach.

Consequently, several of the most recent CCAs, including those in Australia, Canada, and the United Kingdom, have adopted the reserve bidders approach.

C. *Assignment Stage*

In a traditional spectrum auction, if six nearly identical frequency blocks were offered, the seller would ask bidders to submit bids on the specific licenses A to F. A more economical approach is for bidders to indicate the quantity of blocks they would like to purchase during the main part of the auction. Only after the winning quantities have been determined do bidders need to submit bids for specific frequencies. This approach both speeds the progress of the auction and reduces the complexity of bidding.

The main insights behind introducing the CCA's assignment stage were: (i) it is preferable to replace any administrative decisions about the assignment of specific licenses with a bidding process; but (ii) bidding options should be limited only to assignments that would be considered as outcomes of administrative decisions. All other things being equal, a contiguous assignment is considered to create greater value than a checkerboard assignment. Consequently, the assignment stage is limited to assignments where all winners receive contiguous spectrum within a region.

The first CCA with an assignment stage was the 2005 Trinidad and Tobago Spectrum Auction. Essentially all CCAs to date have adopted this approach.

II. Future Evolution of the CCA

This section proposes and discusses three evolutionary enhancements for future CCAs.

A. *Bidding Language*

One of the critical elements of the CCA design is the explicit use of package bids. The initial CCA proposal used a bidding language under which all bids are treated as mutually exclusive ("XOR bids"). The XOR bidding language is understood to be fully expressive, but non-compact.[5]

Until recently, the compactness issue was not a practical concern in the CCA context, since most CCAs were used to allocate relatively small numbers of items. This has changed with the CCA's adoption in countries with regional licenses, such as Australia and Canada. With sufficiently many items, XOR bids might prevent bidders from expressing their values for the relevant packages.

The issue of compactness of bidding language has been studied extensively in the context of combinatorial sealed-bid auctions.[6] One of the main prescriptions is the use of non-mutually-exclusive bids ("OR bids"). More generally, permitting flexible combinations of OR and XOR bids provides multiple ways to improve compactness. In the CCA context, the natural course of evolution is to incorporate a flexible bidding language directly into the supplementary round.

The CCA format presents a somewhat novel environment for designing an effective bidding language. Frequently, a CCA bidder views the supplementary round as an opportunity to express its marginal values for incremental items relative to its final clock package. Furthermore, conditional on winning its final clock package, the bidder may view the incremental items as being locally additive in value. Using this insight, one natural way to introduce OR bidding flexibility into the CCA is to allow a bidder to specify various OR bids as increments on top of its bid for the final clock package.

In conjunction with the compact bidding language, it is important to provide bidders with various controls that they may exercise over their submitted OR bids. For example, a bidder can be allowed to specify a total size limit or a total budget limit for the collection of OR bids that will be considered.

[5] A bidding language is said to be fully expressive if it can be used to communicate any valuation profile. While XOR bids are fully expressive, an astronomical number of XOR bids can be required to express even very simple preferences.

[6] See Nisan (2006) for a review of the literature on bidding language in combinatorial auctions.

The integration of OR bids into the CCA is not completely straightforward. One (perhaps the greatest) challenge is to design an appropriate activity rule for OR bids. Typical CCA activity rules are formulated in terms of whole packages, and are not trivially extended to handle OR components.[7] This issue creates a need for strong and robust activity rules, which can be consistently applied both to XOR and OR bids, such as the activity rule proposed in the next subsection.

B. *Revealed-Preference Activity Rules*

The most fundamental innovation of the SMRA, when it was introduced for spectrum auctions 20 years ago, was the imposition of activity rules on bidders. Activity rules are intended to prevent "bid sniping": the phenomenon widely observed in auctions such as eBay where bidders conceal their true intentions until the very end of the auction. Bid sniping effectively converts a dynamic auction into a sealed-bid auction and thereby works at cross-purposes to a dynamic auction.

Standard implementations of activity rules in SMRAs are based upon "points." The auctioneer assigns a number of points to each item in the auction, most commonly based upon the population of a license region or some other measure of value.[8] The activity rule is then a variant on the requirement that the bidder's total points bid must be non-increasing as the auction progresses. Thus, it requires bidders to submit serious bids in early rounds of the auction in order to retain the right to submit bids for an equivalent quantity (as measured by points) in later rounds.

Point-based activity rules are too weak, in that they create a number of opportunities for strategic bidding. Most notoriously, points give rise to "parking": placing bids on items that one is not interested in buying, for the purpose of stockpiling points for future use.

Less appreciated is the fact that point-based activity rules are, in other respects, too strong. For any choice of points, there exist valuations and price histories such that the point-based activity rule prevents the bidder from bidding straightforwardly according to its values.

Motivated by these considerations, the original CCA proposal suggested incorporating the Weak Axiom of Revealed Preference (WARP) into the activity rule. Nonetheless, the initial implementations of the CCA, as well as recent applications in the United Kingdom and Slovakia, used point-based activity rules.[9]

In a current working paper (Ausubel and Baranov 2014), we propose introducing activity rules based upon the Generalized Axiom of Revealed Preference (GARP), while completely eliminating any role for points. Use of GARP had apparently been overlooked by prior researchers, on account that WARP already appeared to be quite strict and to risk leaving bidders in "dead-ends" (i.e., situations where the only legal next bid is to drop out of the auction). But it turns out that imposing the stricter activity rule may actually be doing bidders a favor by preventing them from getting into trouble in the first place. A WARP dead-end is possible after any bidding history of nonzero bids only if the bidding history itself contains a GARP violation.

The *GARP-based activity rule* is the requirement that, after a price and bid history $(p_1, x_1), \ldots, (p_{t-1}, x_{t-1})$, the bidder is permitted to bid x_t in round t if and only if the history $(p_1, x_1), \ldots, (p_t, x_t)$ is consistent with GARP.

Ausubel and Baranov (2014) prove that the GARP-based activity rule always permits truthful bidding and guarantees that the bidder will never reach a "dead-end."

Most auction research assumes quasilinear bidder values. The above results hold irrespective of whether bidders' values are restricted to be quasilinear. However, a pure GARP-based activity rule (without quasilinearity) appears too weak for practical purposes; in practice, one would probably require consistency with both GARP and quasilinearity.

[7] Given the potentially vast number of values communicated through OR bids, a sensible approach would be to move validation of the activity rule from the bid entry process to the winner determination process. Bids for individual OR components would be entered subject to the usual activity rules, but bids for combinations of OR bids would be capped automatically by the solver (rather than being validated and possibly rejected at the time of bid entry).

[8] For example, a New York license might be assigned 100 points, whereas a Washington, DC license might be assigned 25 points.

[9] Several other recent CCAs (Ireland, Australia, and Canada) have utilized hybrid activity rules based upon both points and WARP.

Such activity rules also are computationally practical. Algorithms are known for validating against GARP (with or without quasilinearity) that are polynomial in the size of the data.

Ausubel and Baranov (2014) also raise the possibility of relaxations or refinements of the quasilinear GARP-based activity rule. Since budget constraints could cause inconsistent bidding, a reasonable relaxation may be to admit the possibility of budget constraints, but otherwise to impose full quasilinear GARP. At the same time, the Strong Axiom of Revealed Preference (SARP) may work as a refinement that deters some strategic manipulations.

C. *Iterative Pricing*

Most dynamic auction formats, both in the literature and in real-world applications, could be characterized as "iterative first-price" auctions. Consider, for example, the English auction for a single item. While it is often modeled as a sealed-bid second-price auction, bidders submit bids which, if they turn out to win, specify the actual amounts that will be paid. While effectively a second-price auction, it is *literally* an iterative first-price auction.

The same statement also applies to the SMRA and most other dynamic formats. However, the CCA is different. Bidders' clock round bids and supplementary bids are entered into winner determination and pricing problems, and the winners' payments may be substantially lower than the nominal amounts of their bids. The CCA as it has evolved today is an iterative second-price auction.

In the CCA, there exists a tension between strict activity rules and second pricing. With strict activity rules and no undersell, bidders are guaranteed to win their packages from the final clock round. In such scenarios, bidders lack incentives to place supplementary bids for winner determination purposes. At the same time, there are minimal consequences to a bidder who inflates its expressed opportunity cost for pricing purposes. Thus, the resulting prices may be either too low or too high.

In Ausubel and Baranov (2014), we propose to evolve the CCA further by transforming it in the direction of an iterative first-price auction. We proceed by asking the question: Given opponents' bids and given the activity rule, what is the maximum amount that a bidder could ever have to pay to win a package if this were the final clock round of the auction? We refer to the answer as the "exposure calculation." Generally, the highest possible payment is less than the nominal bid.

The nominal bid amount for a package could be discounted, based upon the bidder's exposure calculation. Furthermore, one could then utilize the discounted bid amounts for pay-as-bid pricing. In principle, these changes would convert the CCA from an iterative second-price to an iterative first-price auction.

There are some clear advantages in evolving to an iterative first-price approach. With second pricing, supplementary bids serve both an allocation and a pricing role, giving rise to the internal tension within the CCA. With first pricing, attention is focused on the allocation role. The change would also tend to reduce the importance of bidders' budget constraints.

At the same time, there may be other potential approaches besides first pricing to resolve the tensions within the CCA. As noted above, the current CCA design assures a very high degree of stability and predictability in going from the clock rounds to the supplementary round. The stability may be seen as so great that, in some scenarios, second pricing becomes problematic.

Alternatively, one could try to reduce any excessive stability in the current design. This stability has two sources: the strict activity rules, which limit bidders' latitude in placing supplementary bids; and the absolute (rather than relative) interpretation of bids from the clock rounds. When clock bids are taken at face value and supplementary bids are constrained, the winning bids will tend to come from the clock rounds. However, to the extent that clock bids are discounted, there is greater scope for the supplementary bids to change the outcome.

Finally, one should not lose sight that there are advantages to ensuring stability between the clock rounds and supplementary round. When the rules provide stability, bidders have the greatest incentive for truthful bidding in the clock rounds, and the dynamic auction process is the most informative. By contrast, when the outcome can change substantially in the supplementary round, bid sniping becomes effective, the clock rounds lose meaning, and the auction effectively becomes "sealed bid."

III. Conclusion

In this paper, we have proposed three enhancements for future CCAs. Introducing an OR bidding language in the supplementary round would be the most incremental change. In January 2014, the Canadian Government announced that it will adopt this enhancement in its upcoming 2,500 MHz auction. The need is quite clear, as with 318 licenses, grouped into 106 categories, this will be the largest CCA in scale to date.

Incorporating a GARP-based activity rule would also be a modest evolutionary step, as revealed-preference considerations have already been used. Transforming the CCA, from an iterative second-price to an iterative first-price auction, would be the largest step.

Current activity rules may lead to exposure calculations that equal or exceed the final clock prices, while GARP-based activity rules yield exposures more in line with opportunity-cost pricing. Thus, the new activity rule is not only a compelling evolutionary change on its own, but it also facilitates the evolution of the CCA to an iterative first-price auction.

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