An Enhanced Combinatorial Clock Auction*

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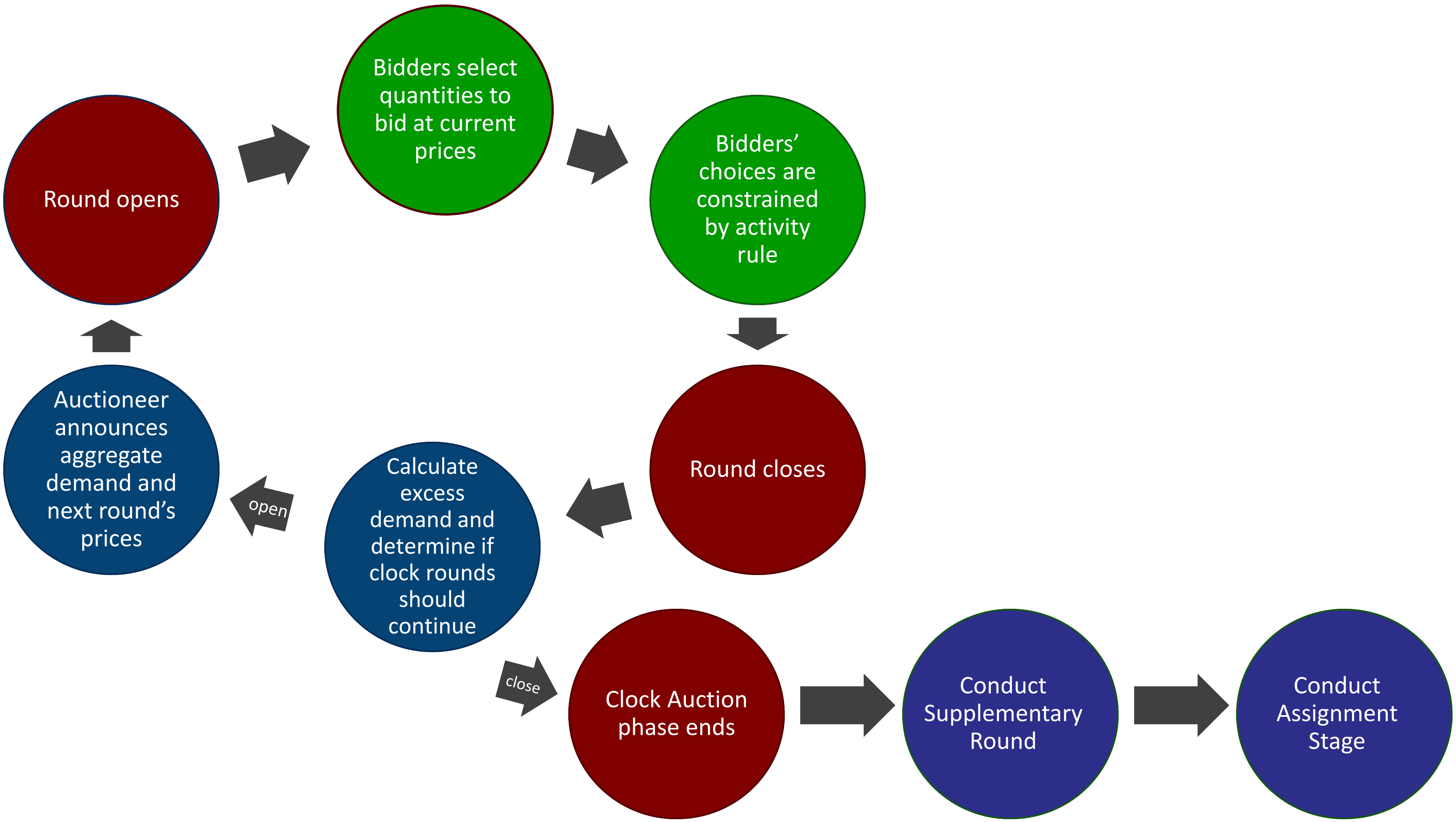
Combinatorial Clock Auction (current generation)

- **Has been used for digital dividend auctions in Denmark, Ireland, the Netherlands, Switzerland and the UK**
- **Has been adopted for upcoming spectrum auctions with regional licenses in Australia and Canada**
- **In the past couple of years, CCAs have been used for more major spectrum auctions worldwide than any other auction format**

Combinatorial Clock Auction (current generation)

- A three-phase auction with package bidding, consisting of a clock auction phase, followed by a supplementary round, and concluding with an assignment round
- Clock auction phase: Auctioneer announces prices and bidders respond with quantities; bidders are constrained by eligibility points and/or a WARP-based activity rule
- Supplementary round: Bidders can supplement their bids from the clock rounds with additional package bids, constrained by a WARP-based activity rule
- Assignment stage: generic spectrum mapped to frequencies
- Allocation and pricing are given by a core-selecting mechanism

Combinatorial Clock Auction process



Critiques of the current generation CCA

- **Most dynamic auction formats are "iterative first-price"— they are *literally* first-price (while *effectively* second-price)**
- **For example, in the English auction (as well as the SMRA, the clock auction and the ascending proxy auction), bids are submitted which, if they turn out to win, specify the amounts that will actually be paid**
- **However, the CCA as used for spectrum auctions is closer to being "iterative second-price"**

Critiques of the current generation CCA

- **This creates some basic tensions in the CCA, as far as stability between the clock stage and supplementary rounds, on the one hand, and price determination, on the other**
  - **With a final price cap and without undersell, each bidder is guaranteed to win its final clock package;**
  - **Bidders may lack incentive to submit supplementary bids;**
  - **Since the price determination depends on supplementary bids, prices may be too low (“missing bids”) or too high (“spiteful bidding”)**
- **It also might exacerbate the effects of budget constraints**

Activity rules in the SMRA and current generation CCA

- **Activity rules are intended to prevent "bid sniping": a bidder must submit bids in early rounds of the clock stage in order to be allowed to continue to submit bids in later rounds (and in the supplementary round)**
- **SMRAs have required monotonicity in eligibility points (and some CCAs have taken a similar approach)**
- **But this approach bumps up against the following result:**
  - **<u>Theorem</u>: For any choice of eligibility points, there exist bidder valuations and price histories such that the bidder is prevented from bidding its true valuations by an activity rule requiring monotonicity in eligibility points; and if bidders attempt to bid straightforwardly, the outcome will necessarily be inefficient**

Critiques of the current generation CCA

- **The incorporation of revealed-preference considerations is an improvement on pure eligibility-point-based activity rules, but current versions are imperfect**
  - **Only some of the revealed-preference constraints are imposed, in order to avoid "dead-ends"**
  - **This creates potential anomalies (e.g., non-monotonicities may be present in the caps)**
  - **Finally, it may be possible for a bidder to influence which revealed-preference constraints are imposed**

Activity rules in the SMRA and current generation CCA

- **Some CCA rules (Ireland, Australia, Canada) have incorporated revealed preference considerations**
  - **WARP: $(p_s - p_t) \bullet (x_s - x_t) \leq 0$, for $s < t$**
  - **Verbally, bids should only involve switches to packages that have become relatively less expensive**
- **A "hybrid" revealed-preference / eligibility-point approach—a bidder can bid on package *Q* if:**
  - **Package *Q* is consistent with revealed preference, with respect to all prior eligibility-reducing rounds; or**
  - **Package *Q* is within the bidder's current eligibility points**

Part I:
Activity Rules

Generalized Axiom of Revealed Preference (GARP)

- **Generalized Axiom of Revealed Preference (GARP):**
  - **Given price-quantity pairs $(x_1, p_1), \ldots, (x_t, p_t)$, package x_i is said to be <u>revealed preferred</u> to x_m if there is a sequence $j, k, \ldots, l$ such that $p_i x_i \geq p_i x_j$, $p_j x_j \geq p_j x_k$, … , $p_l x_l \geq p_l x_m$.**
  - **The data $(x_1, p_1), \ldots, (x_t, p_t)$ are said to <u>satisfy GARP</u> if:**

    x_i is revealed preferred to x_m $\Rightarrow$ $p_m x_m \leq p_m x_i$.
  - **Intuitively, this requires that there is "cyclical consistency": if package x_i is revealed to be preferred to package x_m through a sequence of decisions, package x_m cannot be revealed to be strictly preferred to the original package x_i.**
- **Afriat's Theorem (1967): A finite set of data is consistent with utility maximization if and only if it satisfies GARP.**

GARP-based activity rules

- <u>Definition 1</u>: **The GARP-based activity rule for the clock rounds is the requirement that, given bid history $(x_1, p_1), \ldots, (x_{t-1}, p_{t-1})$, the bidder is permitted to bid x_t in round t if and only if the data $(x_1, p_1), \ldots, (x_t, p_t)$ is consistent with GARP.**
- **This is a stronger activity rule than requiring WARP to hold in every period, which in turn is a stronger activity rule than is used in today's CCAs—see the example on the next slide.**

Example using WARP-based activity rules

- **Consider:**

Round	Price vector	Bidder's demand
1	$p_1 = (1, 1, 1)$	$x_1 = (1, 0, 0)$
2	$p_2 = (1, 1, 4)$	$x_2 = (0, 1, 0)$
3	$p_3 = (4, 1, 4)$	$x_3 = (0, 0, 1)$
4	$p_4 = (4, 3, 5)$	$x_4 =$???

(a) The bidding and WARP have produced a "dead-end": the only allowable bid in Round 4 under WARP is (0, 0, 0).

(b) However, the reason for the dead-end is that the bid in Round 3 already violated GARP.

(c) Under a GARP activity rule, x_3 is not allowed to be bid.

GARP-based activity rules

- **<u>Definition 1</u>: The GARP-based activity rule for the clock rounds is the requirement that, given bid history $(x_1, p_1), \ldots, (x_{t-1}, p_{t-1})$, the bidder is permitted to bid x_t in round t if and only if the data $(x_1, p_1), \ldots, (x_t, p_t)$ is consistent with GARP.**
- **Notwithstanding the strength of the activity rule:**
  - **<u>Proposition 1</u>: Straightforward bidding is always permitted by the GARP-based activity rule.**
  - **<u>Proposition 2</u> (No dead-ends): At least one of the bidder's prior bids, $x_1, \ldots, x_{t-1}$, is always permitted in round t by the GARP-based activity rule.**
  - **<u>Proposition 3</u> (Smaller bids): If bidding x in round t is allowed by the GARP-based activity rule, then bidding $y \le x$ is also allowed.**

Activity rule for the supplementary round

- **<u>Definition 2</u>: Revealed preference for package *x* with respect to round *t* is the following inequality: $b(x) \le b(x_t) + p_t(x - x_t)$.**
- **<u>Definition 3</u>: The activity rule for the supplementary round is the requirement that each supplementary bid is required to satisfy revealed preference with respect to <u>all</u> clock rounds:**

$$b(x) \le CAP(x) \equiv \min_{t=1,\ldots,T} \left\{ b(x_t) + p_t(x - x_t) \right\} .$$

Note: A system of revealed-preference constraints is only defined up to a constant. So, as in the current CCA, a bidder could increase the bid on its final clock package and raise all other bidders in tandem.

Activity rule for the supplementary round

- <u>Definition 4</u>: For any package *x*, let $d_T(x)$ be the maximum violation of the GARP constraints for the package *x* relative to the final clock round *T*:

$$d_T(x) \equiv \max_{\{k_1, \ldots, k_m\} \subset \{1,\ldots,T\}} \left\{ p_T(x - x_{k_1}) + p_{k_1}(x_{k_1} - x_{k_2}) + \ldots + p_{k_m}(x_{k_m} - x) \right\}.$$

- <u>Proposition 6</u>: The highest bid amount possible for a package *x* relative to the bid on the final clock package is:

$$\overline{CAP}_T(x) \equiv p_T x - d_T(x).$$

- <u>Proposition 8</u> (monotonicity): $\overline{CAP}_T(y) \le \overline{CAP}_T(x)$ if $y \le x$.

- <u>Proposition 9</u>: $\overline{CAP}_T(x)$ is a concave function.

Part II:
Pricing

What is the maximum bid that a bidder needs to win?

- **Suppose that the clock stage would end in Round *T* with no undersell, i.e., $x_T^i + X_T^{-i} = S$. Then bidder *i* is assured of winning x_T^i if her bid is at least the maximum combined cap of all opponents for *S*. Define this quantity as $p_T^i(x_T^i)$, where:**

$$
\begin{aligned}
p_T^i(x_T^i) &= \max_{\sum_{j\neq i} x^j = S} \left[\sum_{j\neq i} \overline{CAP_T^j}(x^j)\right] - \sum_{j\neq i} p_T x_T^j \\
&= \max_{\sum_{j\neq i} x^j = S} \left[\sum_{j\neq i} p_T x^j - d_T^j(x^j)\right] - p_T X_T^{-i} \\
&= p_T S - \min_{\sum_{j\neq i} x^j = S} \left[\sum_{j\neq i} d_T^j(x^j)\right] - p_T X_T^{-i} \\
&= p_T x_T^i - \min_{\sum_{j\neq i} x^j = S} \left[\sum_{j\neq i} d_T^j(x^j)\right].
\end{aligned}
$$

What is the maximum bid that a bidder needs to win?

- **The previous slide assumed knowledge of the disaggregated demands. If only the aggregate demand is observable, then bidder *i* is assured of winning x_T^i if her bid is at least the maximum combined cap of an "aggregate opponent" for *S*. Define this quantity as $P_T^i(x_T^i)$. Surprisingly, $P_T^i(x_T^i)$ can be calculated by viewing the aggregate demand as coming from a single player and calculating that player's maximum GARP violation from bidding for *S* in round *T*:**

$$P_T^i(x_T^i) = p_T x_T^i - \Delta_T^i(S), \text{ where:}$$

$$\Delta_T^i(S) \equiv \max_{\{k_1,\ldots,k_m\} \subset \{1,\ldots,T\}} \left\{ p_T(S - X_{k_1}^{-i}) + p_{k_1}(X_{k_1}^{-i} - X_{k_2}^{-i}) + \ldots + p_{k_m}(X_{k_m}^{-i} - S) \right\}.$$

What is the maximum bid that a bidder needs to win?

- **Theorem**:

  (a) **If opponents' clock bids are taken to be linear prices, if the GARP constraints are enforced against them, and if there is no undersell in the final clock round, then:**

  (i) **the bidder's final is guaranteed to be allocated its tentative allocation at the end of the clock stage; and**

  (ii) **if the bidder's bids are equally discounted by a constant $\leq \Delta_T^i(S)$, the bidder is still guaranteed to win x_T^i .**

  (b) **If bidders' clock bids x_T^i are taken to be linear prices discounted by $\Delta_T^i(S)$, the results of part (a) still hold.**

- **Implication**: **This enables us to convert the CCA into an iterative first-price auction!**

What is the maximum bid that a bidder needs to win?

- **Technical point:**

  (1) In the slide three above, it appears necessary, in defining $p_T^i(x_T^i)$, i.e. based on disaggregated demands, to solve the relaxed problem in which fractional pieces of the items can be demanded.

  (2) However, in the slide two above, in defining $P_T^i(x_T^i)$, i.e. based on aggregate demand, there is no need to further relax the problem.

The CCA as an iterative first-price auction

- In the current CCA design, there is a tension between the final price cap and the (core-selecting mechanism) pricing rule:
  - With a final price cap and without undersell, each bidder is guaranteed to win its final clock package;
  - Bidders may lack incentive to submit supplementary bids;
  - Since the price determination depends on supplementary bids, prices may be too low ("missing bids") or too high ("spiteful bidding")
- With first-pricing of the final clock packages, it doesn't matter if there is insufficient incentive for supplementary bids
- Still would probably want to use a core-selecting mechanism to price the "undersell"—this will be included in the paper

The CCA as an iterative first-price auction

- **A strong, revealed-preference-based activity rule assures a high degree of stability and predictability in going from the clock stage to the supplementary round**
- **Moreover, first-pricing would tend to limit the opportunities for strategic bidding in the clock stage**
- **Together, we should expect to see a marked improvement in <u>price discovery</u> in the CCA**
- **The use of first-pricing would also tend to reduce the importance of budget constraints**
- **However, an alternative to first-pricing is simply to impose the GARP constraints and to do price determination using the caps instead of the actual submitted supplementary bids**

Conclusion: Applicability for the Incentive Auction?

- **CCA has been generally successful, so is worth thinking about**
- **Two aspects of the CCA are partially incorporated into the “straw man” design—a clock auction and the use of generic lots/assignment stage**
- **There are limitations to the current CCA design—this paper proposes a next-generation design without these limitations**
- **The use of 176 EA regions would mandate modifications to the standard CCA (and many other combinatorial designs)**
- **While the GARP-based activity rule seems almost perfect for a combinatorial auction, it may not interact so well with the “straw man” clock auction (or with the SMRA), as bidders cannot freely withdraw when there is no excess demand**